Annualised Percentage Rates (APR's)

Most interest rates are quoted as Annualised Percentage Rates (APR's). This is both by convention and in some countries by law. This is true for credit card rates, mortgage rates, bond yields, and many others. APR's are sometimes called nominal rates, but nominal has another meaning related to inflation so we will avoid calling APR's nominal rates.

The compounding period of an APR is usually not explicitly stated. But usually it can be assumed that **the compounding frequency of an APR is the same as the payment frequency**.

For example, a credit card might advertise an interest rate of 24% pa. This must be an APR since all advertised rates have to be APR's by law. Because credit cards are always paid off monthly, the compounding frequency is per month. Therefore the interest rate is 24% pa given as an APR compounding monthly.

While APR’s are the rate that you see quoted and advertised, unfortunately they **cannot** be used to find present or future values of cash flows. You must convert the APR to an effective rate before doing financial mathematics.

Effective Rates

Effective rates compound only once over their time period, and the time period can be of any length, not necessarily annual.

Effective rates can be used to discount cash flows.

APR's **cannot** be used to discount cash flows, they must be converted to effective rates first.

Note that all of the calculation examples up to here have assumed that the interest rate given is an effective rate.

Calculation Example: Present Values and Effective Rates

Question: What is the present value of receiving $100 in one year when the effective monthly rate is 1%?

Answer: Since the effective interest rate is per month, the time period must also be in months, so

$$V_o = \frac{C_t}{(1+r)^t}$$

$$= \frac{100}{(1+0.01)^{12}}$$

$$= 88.7449$$

APR's and Effective Rates

- An APR compounding monthly is equal to 12 multiplied by the effective monthly rate.

$$r_{APR\ comp\ monthly} = r_{eff\ monthly} \times 12$$

- An APR compounding semi-annually is equal to 2 multiplied by the effective 6 month rate.

$$r_{APR\ comp\ per\ 6mths} = r_{eff\ 6mth} \times 2$$

- An APR compounding daily is equal to 365 multiplied by the effective daily rate.

$$r_{APR\ comp\ daily} = r_{eff\ daily} \times 365$$

Calculation Example: Future Values with APR's

Question: How much will your credit card debt be in 1 year if it's $1,000 now and the interest rate is 24% pa?

Answer: Since credit cards are paid off per month, the 24% must be an APR compounding monthly. Therefore the effective monthly rate will be the APR divided by 12.

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12} = \frac{0.24}{12} = 0.02$$

$$V_t = C_0(1+r)^t$$

$$= 1000(1+0.02)^{12} = 1{,}268.2418$$

Converting Effective Rates To Different Time Periods

Compounding the rate higher (to a longer time period):

$$r_{eff\ annual} = \left(1 + r_{eff\ monthly}\right)^{12} - 1$$

$$r_{eff\ semi-annual} = \left(1 + r_{eff\ monthly}\right)^{6} - 1$$

$$r_{eff\ quarterly} = \left(1 + r_{eff\ monthly}\right)^{3} - 1$$

Compounding the rate lower (to a shorter time period):

$$r_{eff\ monthly} = \left(1 + r_{eff\ annual}\right)^{\frac{1}{12}} - 1$$

$$r_{eff\ daily} = \left(1 + r_{eff\ annual}\right)^{\frac{1}{365}} - 1$$

Calculation Example: Converting Effective Rates

Question: A stock was bought for \$10 and sold for \$15 after 7 months. No dividends were paid. What was the effective annual rate of return?

Answer:

First we find the return over 7 months. This will be the effective 7 month rate of return. Note that the time period is in 7-month blocks, so t=1:

$$V_0 = \frac{V_t}{(1+r)^t}$$

$$V_0 = \frac{V_1}{(1+r)^1}$$

$$10 = \frac{15}{(1+r)^1}$$

$$(1+r)^1 = \frac{15}{10}$$

$r = \frac{15}{10} - 1 = 0.5 = 50\%$, which is the effective 7 month rate.

Now we need to convert it to an effective annual rate (EAR).

This can be done by 'compounding up' by 12/7 in one step:

$$r_{eff\ annual} = \left(1 + r_{eff\ 7mth}\right)^{\frac{12}{7}} - 1$$

$$= (1 + 0.5)^{12/7} - 1 = 1.0039 = 100.39\%$$

Or it can be broken down into two steps:

- Compounding the 7-month rate down to a monthly rate:

$$r_{eff\ monthly} = \left(1 + r_{eff,7mth}\right)^{1/7} - 1$$
$$= (1 + 0.5)^{1/7} - 1 = 0.059634 = 5.9634\%$$

- Then compound the monthly rate up to a 12-month (annual) rate:

$$r_{eff\ annual} = \left(1 + r_{eff,monthly}\right)^{12} - 1$$
$$= (1 + 0.059634)^{12} - 1 = 1.0039 = 100.39\%$$

Calculation Example: Converting APR's to Effective Rates

Question: You owe a lot of money on your credit card. Your credit card charges you 9.8% pa, given as an APR compounding per month.

You have the cash to pay it off, but your friend wants to borrow money from you and offers to pay you an interest rate of 10% pa given as an effective annual rate.

Should you use your cash to pay off your credit card or lend it to your friend?

Assume that your friend will definitely pay you back (no credit risk).

Answer:

Since the loan interest rate is an effective rate but the credit card rate is an APR we can't compare them.

Let's convert the credit card's 9.8% APR compounding per month to an effective annual rate:

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12}$$

$$= \frac{0.098}{12} = 0.0081667$$

$$r_{eff\ annual} = \left(1 + r_{eff\ monthly}\right)^{12} - 1$$

$$= (1 + 0.0081667)^{12} - 1 = 0.1025 = 10.25\%$$

So the credit card's 9.8% APR compounding per month converts to an effective annual rate of 10.25%. This is more than the loan's 10% effective annual rate, so you should pay off your credit card instead of lending to your friend.

Questions: APR's and Effective Rates

http://www.fightfinance.com/?q=290,330,16,26,131,49,64,265,

Calculation Example: Fully Amortising Loans

Question: Mortgage rates are currently 6% and are not expected to change.

You can afford to pay $2,000 a month on a mortgage.

The mortgage term is 30 years (matures in 30 years).

What is the most that you can borrow using a fully amortising mortgage loan? Fully amortising means that the loan will be fully paid off at maturity.

Answer: Since the mortgage is fully amortising, at the end of the loan's maturity the whole loan will be paid off.

The bank will lend you the present value of your monthly payments for the next 30 years. This can be calculated using the annuity formula.

The $2,000 payments are monthly, therefore the interest rate and time periods need to be measured in months too.

$$t = 30 \times 12 = 360\, months$$

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12} = \frac{0.06}{12} = 0.005 = 0.5\%$$

$$V_0 = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$= \frac{2000}{0.005}\left(1 - \frac{1}{(1+0.005)^{360}}\right) = \$333{,}583$$

Questions: Fully Amortising Loans

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