

## ***Example: Optimal Hedge Ratio***

**Question:** Today is 1<sup>st</sup> January. An Australian pig farmer plans to sell pigs on 15<sup>th</sup> January. He's afraid of the pig price falling.

Unfortunately there are:

- No Australian Securities Exchange (ASX) futures on hogs. So he'll have to cross-hedge using United States CME hog futures.
- No US CME hog futures expiring in January. The next one expires on 13<sup>th</sup> February.

Assume the following:

- 40% correlation between Australian hog prices and US CME hog futures prices;
- 30% pa standard deviation of Australian pig prices;
- 15% pa standard deviation of US CME hog futures prices.
- The Australian farmer plans to sell 300,000 pounds of pigs on 15Jan.
- One US CME hog futures contract is on 40,000 pounds of pigs.
- Australian pig prices are \$0.70 per pound on 1Jan.
- US CME hog futures contracts maturing on 13Feb have a futures price of \$0.63 per pound on 1Jan.

**Answer:** The Australian pig farmer should hedge against falls in the pig price by **selling** futures that expire on 13Feb. He should short the futures today (1Jan).

On 15Jan when he sells the pigs, he should then 'close out' the short futures by **buying** lean hog futures contracts that expire on 13Feb.

There will be basis risk caused by cross-hedging and the premature closing out of futures. But the risk can be minimised using the optimal hedge ratio ( $h$ ):

$$h = \rho_{S_T, F_{0,T}} \cdot \frac{\sigma_{S_T}}{\sigma_{F_{0,T}}} = 0.4 \times \frac{0.3}{0.15} = 0.8$$

The hedge ratio can be interpreted as: “A 1% increase in the US CME hog futures price is expected to cause a 0.8% increase in the Australian pig price”.

Calculate the optimal number of futures contracts (n) to buy, which requires tailing since futures have margin accounts:

$$n_{tailing} = h \cdot \frac{V_S}{V_F} = h \cdot \frac{Q_S \cdot S_t}{Q_F \cdot F_{t,T}} = 0.8 \times \frac{300,000 \times 0.7}{40,000 \times 0.63} = 6.67$$

Since fractions of futures contracts can't be bought, round the answer to the nearest whole number of contracts which is 7.

Therefore the Australian pig farmer should short 7 futures contracts on 1Jan, then when he sells his pigs on 15Jan he should long another 7 contracts that also mature on 13Feb to close out his position. The hedge is imperfect since there will be basis risk.