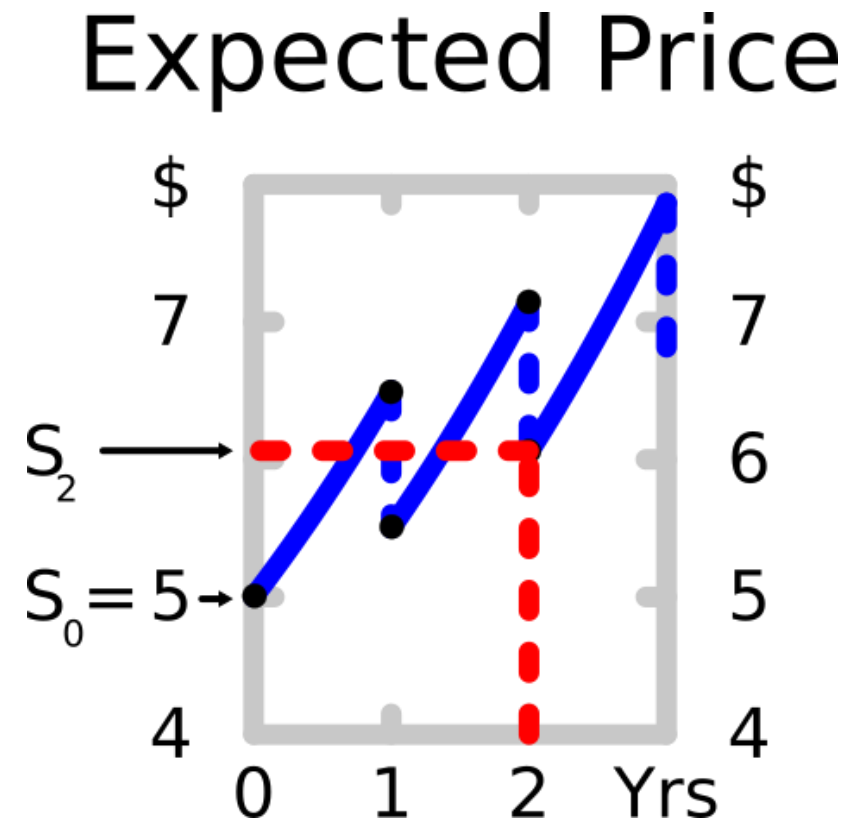


Calculation Example: Futures Price and Payoff at Maturity

Question: Calculate the futures price of a 2 year futures contract written on the following stock. Assume that the futures contract matures just after the stock pays its dividend in year 2.

The stock has a current price of \$5.
Its next annual dividend of \$1 will be paid in one year, and the dividend will continue to be paid annually forever and will grow at 10% pa.

The stock's required total return is 30% pa. The perpetuity equation is



suitable for valuing this share. All rates are given as effective annual returns. There are no storage costs from holding stocks.

Answer: The futures price will be equal to the expected stock price in 2 years since there's no storage costs.

The dividend yield is 20%: $r_{div,eff} = \frac{C_1}{S_0} = \frac{1}{5} = 0.2$

$$F_T = E[S_T] = S_0 \cdot (1 + r_{total,eff} - r_{div,eff} + r_{cost,eff})^T$$

$$\begin{aligned} F_2 = E[S_2] &= 5 \times (1 + 0.3 - 0.2 + 0)^2 \\ &= 5 \times (1 + 0.1)^2 = 6.05 \end{aligned}$$

So the futures price will be \$6.05. So if you agree to buy this futures contract, you will lock in to pay \$6.05 for this stock in 2 years.

Question: Will you receive the dividends at time 1 and 2 if you buy this 2 year futures contract now? Remember that the futures contract matures just after the dividend at time 2.

Answer: No, you will not receive the dividends at time 1 and 2 since the futures contract doesn't mature until after the dividend at time 2 is paid. If the future is physically settled rather than cash settled, then from time 2 onwards you will own the stock and you will be entitled to any dividends.

Question: If the underlying stock price at maturity in two years turns out to be \$5.80, what will be your payoff at maturity from the futures contract?

Answer:

$$f_{T,LF} = S_T - K_T$$

$$\begin{aligned} f_{2,LF} &= 5.80 - 6.05 \\ &= -0.25 \end{aligned}$$

Therefore you will have lost \$0.25 on your long futures contract.

Your counterparty who has the short futures position will have gained \$0.25.