European Option Put-Call Parity

Buying a stock and buying a put option is equivalent to buying a call option and buying government bonds, where both European-style options are written on the same underlying stock, with the same strike price and maturity, and the government bond investment equals the present value of the options' strike price:

LongStock + LongPut = LongCall + LongGovtBonds

$$S_0 \quad + \quad p_0 \quad = \quad c_0 \quad + \quad K_T.\,e^{-r.T}$$

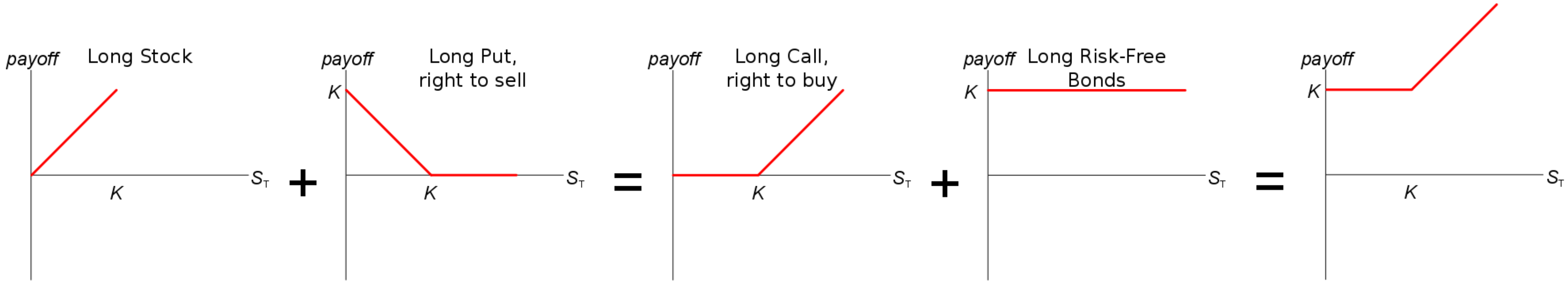
The above equation shows asset prices at time zero (now).

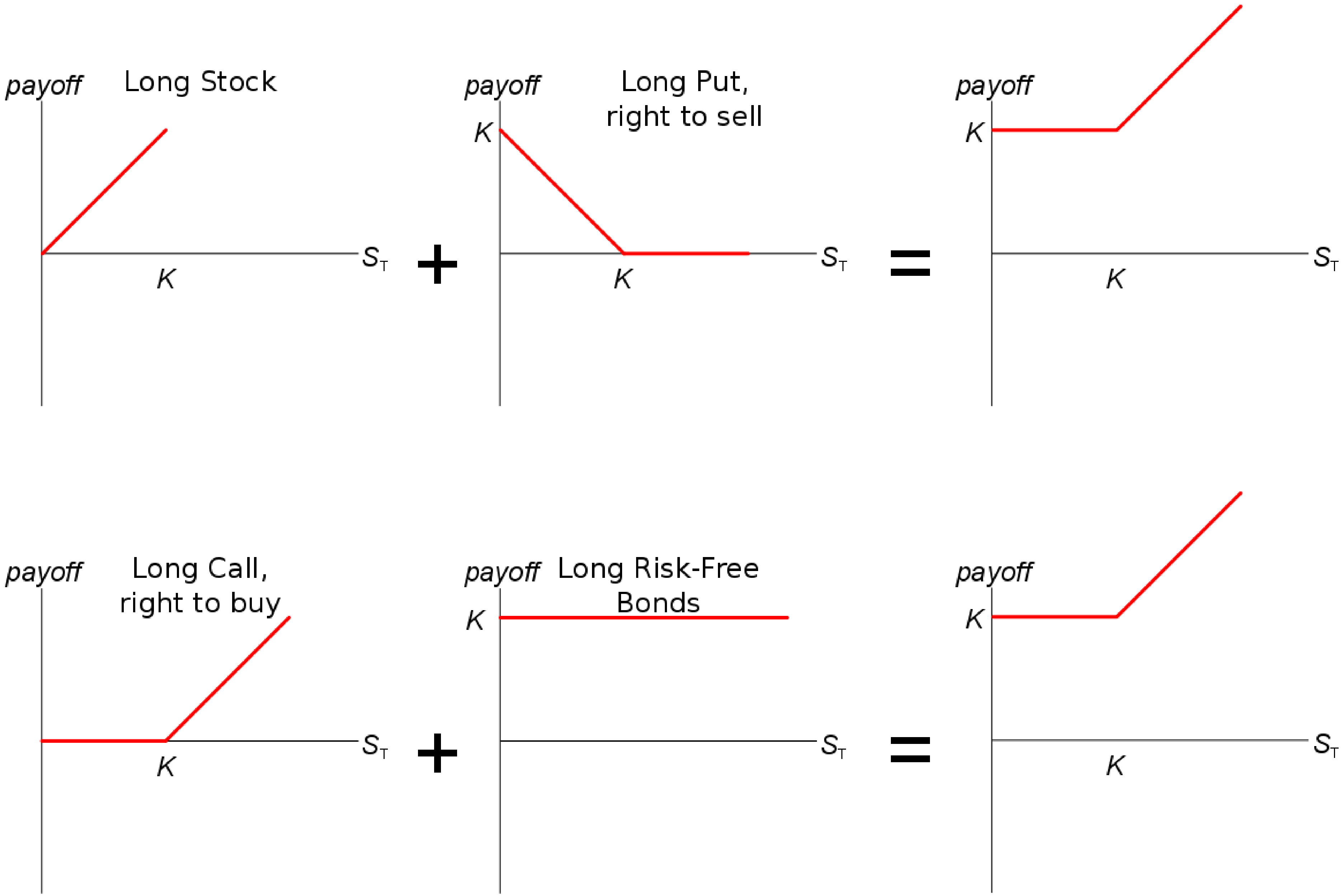
This relationship should always hold before and at maturity. At any time t:

$$S_t + p_t = c_t + K_T.\,e^{-r.(T-t)}$$

At maturity (T):

$$S_T \quad + \quad p_T \quad = \quad c_T \quad + \quad K_T$$




payoff
Long Stock
K
S_T
+
payoff
Long Put,
right to sell
K
K
S_T
=
payoff
K
K
S_T
payoff
Long Call,
right to buy
K
S_T
+
payoff
Long Risk-Free
Bonds
K
S_T
=
payoff
K
K
S_T

Investments versus Cash Flows

A positive investment is a negative cash flow, so put-call parity expressed as an investment now (t=0) uses the familiar equation:

$$S_0 + p_0 = c_0 + K_T . e^{-r.T}$$

But put-call parity expressed as cash flows now (t=0) is:

$$-S_0 - p_0 = -c_0 - K_T . e^{-r.T}$$

This is because investing in, say, a stock, means paying money now which is a negative cash flow now, hence the switch in the signs (+ to -). So:

Buying stock = Long stock = Investing in stock = Spending cash on stock = Negative cash flow on stock purchase

Adjustment for Dividends

Dividends can be included by simply replacing all instances of S_0 with:

- $S_0 - D_0$ where D_0 is the present value of the discrete dividend paid at time t, so $D_0 = D_t . e^{-r.t}$ and r is the continuously compounded risk free rate:
  $(S_0 - D_0) + p_0 = c_0 + K_T . e^{-r.T}$

- $S_0 . e^{-q.t}$ where q is the continuously compounded annual dividend yield, say when S is a stock index such as the S&P500:
  $(S_0 . e^{-q.t}) + p_0 = c_0 + K_T . e^{-r.T}$