Calculation Example: Risk-neutral Valuation

Question: A stock is priced at \$50. In one year the stock price will increase by 20% to \$60 or fall by 10% to \$45. The continuously compounded risk free rate is 10% pa. Value a European call option that expires in one year with a strike price of \$52.

Answer:

1. Calculate the probability of the share price increasing,

$$p = \frac{e^{r.t} - d}{u - d}$$

$$u = 1 + 0.2 = 1.2$$

$$d = 1 - 0.1 = 0.9$$

$$p = \frac{e^{0.1\times1} - 0.9}{1.2 - 0.9} = 0.684$$

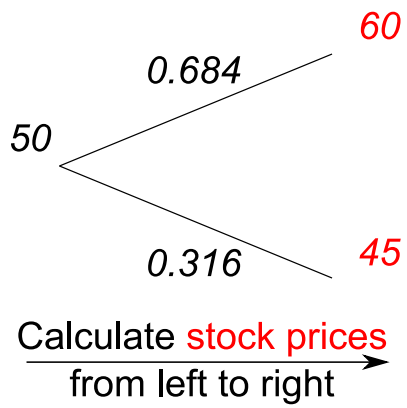
Or alternatively:

$$S_0 = (p.S_{tu} + (1-p).S_{td})e^{-r.t}$$

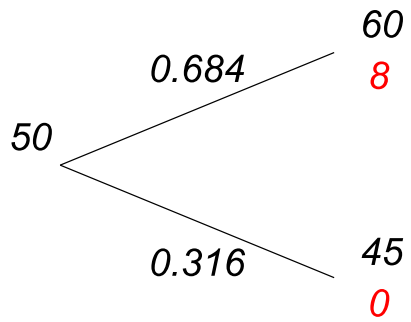
$$50 = (p \times 60 + (1-p) \times 45)e^{-0.1\times1}$$

$$p = 0.684$$

2. Calculate share prices at each node of the tree by multiplying the current share price by the up and down moves, working from left to right.



3. Calculate option values at the leaves of the tree on the very right. Since we are valuing a call,



Calculate final option prices on the far right

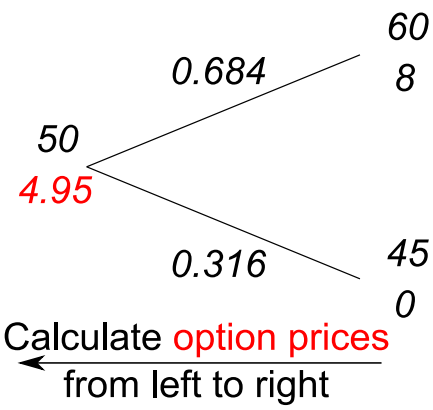
$$f_T = \max(S_T - K, 0)$$
$$f_{1u} = \max(60 - 52, 0) = 8$$
$$f_{1d} = \max(45 - 52, 0) = 0$$

4. Then find the discounted expected value of the option working from right to left, using

$$f_0 = (p.f_{1u} + (1-p)f_{1d})e^{-r.t}$$

$$f_0 = (0.684 \times 8 + (1 - 0.684) \times 0)e^{-0.1\times 1}$$

$$= 4.95$$



So the value of the call option now, which is the same as its price or premium right now, is \$4.95.

Notice that this is the same answer as the previous example using the no-arbitrage valuation approach.