Valuation of European Options with Discrete Dividends

To value an option written on a stock that pays a discrete dividend D_t at time t, simply replace all instances of S_0 with $(S_0 - \boldsymbol{D_0})$, where D_0 is the present value of the dividend, so $D_0 = D_t . e^{-r.t}$.

$$c_0 = \quad (S_0 - \boldsymbol{D_0}).N[d_1] \quad - K.e^{-rT}.N[d_2]$$

$$p_0 = -(S_0 - \boldsymbol{D_0}).N[-d_1] + K.e^{-rT}.N[-d_2]$$

$$d_1 = \frac{\ln[(S_0 - \boldsymbol{D_0})/K_T] \ + \left(r + \frac{\sigma^2}{2}\right).T}{\sigma.\sqrt{T}}$$

$$d_2 = \frac{\ln[(S_0 - \boldsymbol{D_0})/K_T] \ + \left(r - \frac{\sigma^2}{2}\right).T}{\sigma.\sqrt{T}} = d_1 - \sigma.\sqrt{T}$$

Calculation Example: European Option Valuation with Discrete Dividends

Question: Find the price of a 6 month European call option with a strike price of $50, written on a stock currently trading at $60 which will pay a:

- $1 dividend in 2 months;
- $1 dividend in 5 months; and a
- $1 dividend in 8 months.

The risk-free interest rate is 10% p.a. continuously compounded and the standard deviation of the stock's returns is 20% p.a..

Answer:

Find the present values of the dividends (D_0). Note that we only care about the dividends paid within the life of the option which is 6 months. So the dividend at 8 months can be ignored since it occurs after the option has expired. It's a red herring!

$$D_0 = \frac{D_{2mth}}{e^{r\times 2/12}} + \frac{D_{5mth}}{e^{r\times 5/12}}$$

$$= \frac{1}{e^{0.1\times\frac{2}{12}}} + \frac{1}{e^{0.1\times\frac{5}{12}}}$$

$$= 1.942660911$$

$$d_1 = \frac{\ln[(S_0 - \boldsymbol{D_0})/K_T] + \left(r + \frac{\sigma^2}{2}\right).T}{\sigma.\sqrt{T}}$$

$$d_1 = \frac{\ln[(60 - 1.942660911)/50] + \left(0.1 + \frac{0.2^2}{2}\right) \times 0.5}{0.2 \times \sqrt{0.5}}$$

$$= 1.480739029$$

$$d_2 = d_1 - \sigma.\sqrt{T}$$

$$d_2 = 1.480739029 - 0.2 \times \sqrt{0.5}$$

$$= 1.339317673$$

$$c_0 = (S_0 - \boldsymbol{D_0}).N[d_1] - K_T.e^{-rT}.N[d_2]$$

$$= (60 - 1.942660911) \times N[1.480739029]$$

$$-50 \times e^{-0.1\times0.5} \times N[1.339317673]$$

$$= 58.05733909 \times 0.930661935$$

$$-50 \times e^{-0.1\times0.5} \times 0.909766361$$

$$= 54.03175556 - 43.26982661$$

$$= \$10.76192895$$