Valuation of European Options with Continuous Dividends

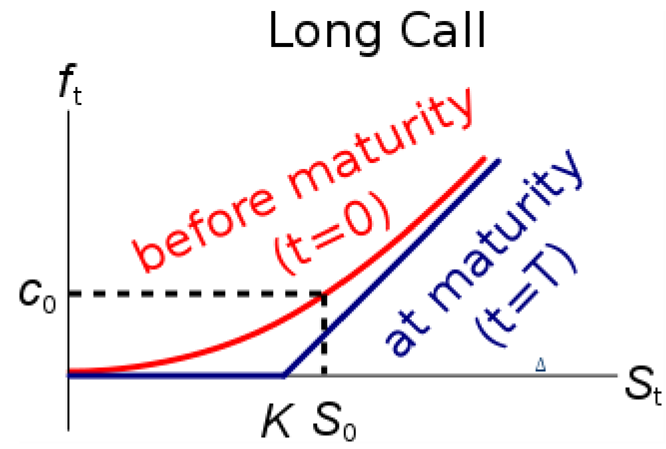
To value an option written on an equity index that pays a continuously compounded dividend yield $\boldsymbol{q}$, simply replace all instances of S_0 with $S_0.\boldsymbol{e^{-q.T}}$:

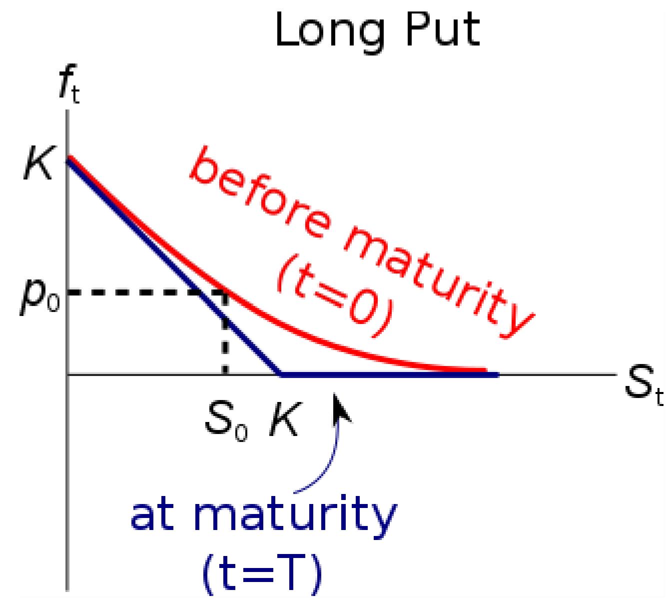
$$c_0 = \quad S_0.\boldsymbol{e^{-q.T}}.N[d_1] \quad - K.e^{-r.T}.N[d_2]$$

$$p_0 = -S_0.\boldsymbol{e^{-q.T}}.N[-d_1] + K.e^{-r.T}.N[-d_2]$$

$$d_1 = \frac{\ln[S_0.\boldsymbol{e^{-q.T}}/K_T] \; + \left(r + \frac{\sigma^2}{2}\right).T}{\sigma.\sqrt{T}}$$

$$d_2 = \frac{\ln[S_0.\boldsymbol{e^{-q.T}}/K_T] \; + \left(r - \frac{\sigma^2}{2}\right).T}{\sigma.\sqrt{T}} = d_1 - \sigma.\sqrt{T}$$





Calculation Example: Valuation of European Index Options with Continuous Dividends

Question: A stock index stands at 4500 points and has a 40% pa standard deviation of returns. The index pays a constant dividend yield of 4% pa and the risk free rate is 10% pa, both with continuous compounding. Value a 3 month European **put** option on the index with a strike of 5000 points.

Answer:

$$d_1 = \frac{\ln[S_0 . \boldsymbol{e}^{-\boldsymbol{q}.\boldsymbol{T}}/K_T] \ + \left(r + \frac{\sigma^2}{2}\right).T}{\sigma.\sqrt{T}}$$

$$d_1 = \frac{\ln[4500 \times \boldsymbol{e}^{-\boldsymbol{0.04 \times 0.25}}/5000] \ + \left(0.1 + \frac{0.4^2}{2}\right) \times 0.25}{0.4 \times \sqrt{0.25}}$$

$$d_1 = -0.351802578$$

$$d_2 = d_1 - \sigma.\sqrt{T}$$

$$d_2 = -0.351802578 - 0.4 \times \sqrt{0.25}$$

$$= -0.551802578$$

$$p_0 = -S_0 . \boldsymbol{e}^{-\boldsymbol{q}.\boldsymbol{T}} . N[-d_1]$$

$$+K . e^{-r.T} . N[-d_2]$$

$$p_0 = -4500 . \boldsymbol{e}^{-\mathbf{0.04} \times \mathbf{0.25}} . N[- - 0.351802578] +$$

$$+5000 \times e^{-0.1 \times 0.25} . N[- - 0.551802578]$$

$$= - 2840.235923$$

$$+3459.708022$$

$$= 619.4720993$$