

Question 1: Which of the following derivative instruments has a non-zero value when it is first issued?

- * (a) Call option.
- (b) Futures contract.
- (c) Forward contract.
- (d) Forward rate agreement.
- (e) Interest rate swap.

Question 2: Which of the following derivative instrument positions does **NOT** require a margin account?

- (a) Long futures contract.
- (b) Short futures contract.
- * (c) Long call option.
- (d) Short call option.
- (e) Short put option

Question 3: Which of the following best describes a **long** position in an **American-style put** option?

- (a) The right to buy the underlying asset for the exercise price at the option's exercise date.
- (b) The right to sell the underlying asset for the exercise price at the option's exercise date.
- (c) The right to buy the underlying asset for the exercise price at any time on or before the option's exercise date.
- * (d) The right to sell the underlying asset for the exercise price at any time on or before the option's exercise date.
- (e) The obligation to buy the underlying asset for the exercise price at the option's exercise date.

Question 4: Which of the following statements about fixed-for-floating interest rate swaps is **NOT** correct?

- *(a) The principals are exchanged at the beginning and end of the swap.
- (b) The fixed and floating payments throughout the life of the swap are netted.
- (c) The fixed payments are called the 'fixed leg', and the floating payments are called the 'floating leg' of the swap.
- (d) If the yield curve is normal, then at the beginning of a swap's life the fixed leg payments are expected to be greater than the floating leg payments.
- (e) If the yield curve is normal, then at the end of a swap's life the floating leg payments are expected to be greater than the fixed leg payments.

Question 5: How is the value of a fixed-for-floating swap best calculated? Assume that the bonds mentioned

below on which the swaps are based have \$100 face values and have the same maturity as the swaps.

The value of a swap to the party paying the floating leg is equal to the swap's notional principal divided by \$100, multiplied by, in brackets, the:

- (a) Price of a fixed coupon bond plus the price of a floating rate bond.
- *(b) Price of a fixed coupon bond less the price of a floating rate bond.
- (c) Price of a fixed coupon bond multiplied by the price of a floating rate bond.
- (d) Price of a fixed coupon bond divided by the price of a floating rate bond.
- (e) Face value of a fixed coupon bond plus by the face value of a floating rate bond.

Question 1 (total of 8 marks): A stock index is expected to pay a continuously compounded dividend yield 4% pa for the foreseeable future. The index is currently at **5,000** points and the continuously compounded total required return is 9% p.a.. An investor has just taken a long position in an **8-month** futures contract on the index.

Question 1a (3 marks): Compute the futures price in index points.

$$\begin{aligned}
 *F_{8\text{mth}} &= S_0 * e^{(r-q)*T} \\
 &= 5,000 * \exp((0.09-0.04)*8/12) \\
 &= 5169.475568
 \end{aligned}$$

Question 1b (1 marks): Compute the initial value of the futures contract.

$$*V_0 = 0$$

Question 1c (4 marks): Six months later the index has fallen to **4,900** points and the expected total required return and dividend yields are unchanged.

Compute the new value of the **long** position in the futures contract in index points. Note that the new value of the contract should be found, not the new futures price.

*First find the expected index price at $t=8\text{mth}$.

$$\begin{aligned}
 E(S_{8\text{mth}}) &= S_{6\text{mth}} * \exp((r-q)*2/12) \\
 &= 4,900 * \exp((0.09-0.04)*2/12) \\
 &= 4941.003946
 \end{aligned}$$

Then find the current value of the long future which is the present value of $S_t - K_t$

$$\begin{aligned}
 V_{6\text{mth},\text{long}} &= (E(S_{8\text{mth}}) - K_{8\text{mth}}) / \exp(r*2/12) \\
 &= (4941.003946 - 5169.475568) / \exp(0.09*2/12) \\
 &= -225.0701227 \text{ index points}
 \end{aligned}$$

Question 2 (8 marks): The below table summarises the borrowing costs confronting two companies.

Borrowing Costs		
	Fixed Rate	Floating Rate
Firm A	6%	6-month LIBOR + 2%
Firm B	6%	6-month LIBOR + 2.4%

Note that they can both borrow fixed at 6% pa, but the floating rates are different.

Suppose Firm A wants to borrow at a fixed rate and Firm B wishes to borrow floating.

Design an intermediated swap that provides a bank with a spread of **8** basis points p.a., and divides the remaining swap benefits **equally** between the two companies.

Use a clearly labelled diagram to summarise the terms of the arrangement.

*Neither firm has an absolute advantage in the fixed rate market, but firm A is better in the floating rate market.

Therefore firm A has a comparative advantage in the floating rate market so it should issue a floating rate bond. Firm B has a comparative advantage in the fixed rate market so it should issue a fixed rate bond.

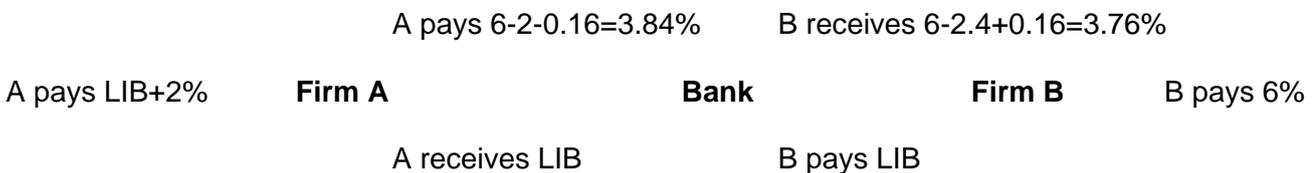
The total benefit available to all 3 parties including the bank is the absolute value of the difference of differences which is:

$$\text{TotalBenefitToABAndBank} = ||6-6| - |(2-2.4)|| = |0 - 0.4| = 0.4\%$$

Subtract the bank's spread top find the benefit to the banks:

$$\text{TotalBenefitToAandB} = 0.4\% - 0.08\% = 0.32\%$$

Firm A and B will share the benefits equally, so 0.16% (=0.32%/2) benefit each.



Question 4 (total of 8 marks): Consider a 6 month European call option with a strike price of \$5, written on a dividend paying stock currently trading at \$5.50. The dividend is paid annually and the next dividend is expected to be \$0.30, paid in 3 months. The risk-free interest rate is 5% p.a. continuously compounded and the standard deviation of the stock's returns is 40% p.a..

Question 4a (3 marks): Calculate d_1 .

$$*d_1 = 0.371008752$$

Question 4b (1 mark): Calculate d_2 .

$$*d_2 = 0.08816604$$

Question 4c (1 mark): Calculate $N(d_1)$ using the tables in the back of this exam paper.

$$*N(d_1) = 0.644684494$$

Question 4d (1 mark): Calculate $N(d_2)$ using the tables in the back of this exam paper.

$$*N(d_2) = 0.535127646$$

Question 4e (2 marks): Calculate the call option price.

$$*c_0 = 0.745185401$$

Question 5 (total of 8 marks): Suppose a stock currently trades at \$100. The stock's semi-annual dividend is expected to be \$6, paid in 3 months from now. A 6-month European call option with a strike price of \$95 has a premium of \$9.83. Assume a 10% continuously compounded risk-free rate.

Question 5a (3 marks): Calculate the price of 6-month European put option with a strike price of \$95 on this stock, as implied by the above information.

$$C_0 + K \cdot e^{-(r \cdot T)} = P_0 + S_0 - D_0$$

$$9.83 + 95/\exp(0.1 \cdot 6/12) = P_0 + 100 - 6/\exp(0.1 \cdot 3/12)$$

$$P_0 = 9.83 + 95/\exp(0.1 \cdot 6/12) - (100 - 6/\exp(0.1 \cdot 3/12))$$

= 6.05 to the nearest cent.

Question 5b (5 marks): If the call option price mentioned above suddenly rose to \$11 but all else was unchanged and there was no news about the company, then explain how you could conduct a risk-free arbitrage. Assume that the call option is mis-priced. You're best able to show the steps using an arbitrage table.

*Short the physical call since it's overpriced. Long the synthetic call (=long put, long stock and short bond) to balance out the risk.

Viewing the below amounts as cash flows at time zero, then all positive cash flows are receipts to us now which are sell (short) transactions and all negative cash flows are payments from us now which are buy (long) transactions:

$$c_0 + K \cdot e^{-r \cdot T} = p_0 + (S_0 - D_0)$$

$$c_0 = p_0 + (S_0 - D_0) - K \cdot e^{-r \cdot T}$$

$$-c_0 = -p_0 - (S_0 - D_0) + K \cdot e^{-r \cdot T}$$

LongSyntheticCall = LongPut + LongStock + ShortBond

To find the amounts of these assets that we need to long and short to make a risk free zero capital arbitrage, we'll use an arbitrage table:

Action	t=0	t=3mth	t=6mth, ST>K	t=6mth, ST<K
Short physical call	11		-(ST-95)	0
Long put	-6.05		0	95-ST
Long stock	-100	6	ST	ST
Short bond to cover dividend (borrow now)	5.8519 (=6/exp(3/12*0.1)) (Step 4)	-6 (Step 3)		
Short bond (borrow now)	90.3668 (=95/exp(6/12*0.1)) (Step 5)		-95 (Step 2)	-95 (Step 2)
Total	1.1687 (Step 6)	0 (Step 1)	0 (Step 1)	0 (Step 1)

Formulas

$$r_{\text{continuously compounded}} = \ln[1 + r_{\text{discrete}}]$$

$$P_0 = \frac{P_t}{e^{t \cdot r_{\text{continuously compounded}}}}$$

$$P_0 = \frac{P_t}{(1 + r_{\text{discrete}})^t}$$

$$r_{\text{discrete}} = e^{r_{\text{continuously compounded}}} - 1$$

$$h^* = \rho_{S,F} \cdot \frac{\sigma_S}{\sigma_F}$$

$$N_{\text{no tailing}}^* = h^* \cdot \frac{Q_S}{Q_F}$$

$$N_{\text{tailing}}^* = h^* \cdot \frac{V_S}{V_F}$$

$$F_{0,T} = S_0 \cdot e^{r \cdot T}$$

$$f_{0, \text{long}} = S_0 - K_T \cdot e^{-r \cdot T}$$

$$f_{\text{long}} = -f_{\text{short}}$$

$$p = \frac{e^{rt} - d}{u - d}$$

$$u = e^{\sigma \sqrt{t}}$$

$$d = \frac{1}{u} = e^{-\sigma \sqrt{t}}$$

$$c_0 + K \cdot e^{-r \cdot T} = p_0 + S_0$$

$$c_0 = S_0 \cdot N[d_1] - K \cdot e^{-r \cdot T} \cdot N[d_2]$$

$$p_0 = -S_0 \cdot N[-d_1] + K \cdot e^{-r \cdot T} \cdot N[-d_2]$$

$$d_1 = \frac{\ln[S_0/K] + (r + \sigma^2/2) \cdot T}{\sigma \cdot T^{0.5}}$$

$$d_2 = d_1 - \sigma \cdot T^{0.5} = \frac{\ln[S_0/K] + (r - \sigma^2/2) \cdot T}{\sigma \cdot T^{0.5}}$$

$$\Delta_{\text{call}} = \frac{\partial c}{\partial S} = N[d_1]$$

$$\Gamma_{\text{call}} = \Gamma_{\text{put}} = \frac{\partial \Delta_{\text{call}}}{\partial S} = \frac{\partial^2 c}{\partial S^2} = \frac{\left(\frac{1}{(2 \cdot \pi)^{1/2}} \cdot e^{-x^2/2} \right)}{S_0 \cdot \sigma \cdot T^{1/2}}$$

$$\Theta_{\text{call}} = \frac{\partial c}{\partial T} = \frac{\left(S_0 \cdot \frac{1}{(2 \cdot \pi)^{1/2}} \cdot e^{-x^2/2} \cdot \sigma \right)}{2 \cdot T^{1/2}} - r \cdot K \cdot e^{r \cdot T} \cdot N[d_2]$$

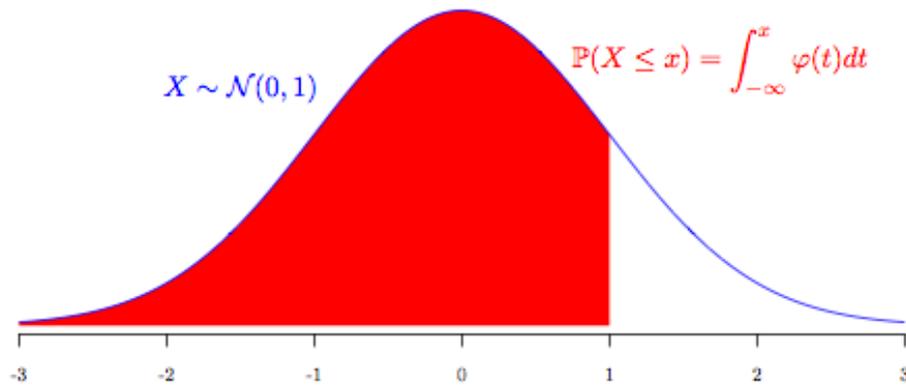
$$\Theta_{\text{put}} = \frac{\partial p}{\partial T} = \frac{\left(S_0 \cdot \frac{1}{(2 \cdot \pi)^{1/2}} \cdot e^{-x^2/2} \cdot \sigma \right)}{2 \cdot T^{1/2}} + r \cdot K \cdot e^{r \cdot T} \cdot N[-d_2]$$

$$c_{t+h} \approx c_t + \epsilon \cdot \Delta_{\text{call}}[S_t] + \frac{1}{2} \cdot \epsilon^2 \cdot \Gamma_{\text{call}}[S_t] + h \cdot \Theta_{\text{call}}[S_t]$$

$$VaR_{\text{prob}} = -V \cdot (\mu + \alpha_{\text{prob}} \cdot \sigma)$$

$$\alpha = \phi^{-1}[1 - \text{prob}] = \text{NormsInv}[1 - \text{prob}]$$

$$ES[\text{prob}] = \mu + \sigma \cdot \frac{\phi[\phi^{-1}[1 - \text{prob}]]}{1 - \text{prob}}$$



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990