

# WESTERN SYDNEY UNIVERSITY

## Final Exam – Spring Session 2015

### School of Business

#### **STUDENT DETAILS**

*Complete your details in this section when instructed by the Exam Supervisor at the start of the exam.*

*You should also complete your details on any answer booklets provided.*

<b>STUDENT SURNAME:</b>	
<b>STUDENT FIRST NAME:</b>	
<b>STUDENT ID:</b>	

#### **EXAM INSTRUCTIONS**

*Read all the information below and follow any instructions carefully before proceeding.*

*You must comply with all directions given by Exam Supervisors.*

*You may begin writing when instructed by the Exam Supervisor at the start of the exam.*

*Clearly indicate which question you are answering on any Examination Answer Booklets used.*

<b>Unit Name:</b>	Derivatives
<b>Unit Number:</b>	200079
<b>Number of Questions:</b>	Part A has 5 questions, Part B has 4 questions.
<b>Total Number of Pages:</b>	7 (excluding exam cover sheet)
<b>Value of Questions:</b>	Part A questions are worth 2 marks each. Part B questions are worth 10 marks each. This gives a total of 50 marks.
<b>Answering Questions:</b>	Answer Part A multiple choice questions on the scan sheet provided Answer Part B short answer questions on the exam paper itself.
<b>Lecturer/Unit Coordinator:</b>	Keith Woodward / Maria Varua
<b>Time Allowed:</b>	2 hours

#### **RESOURCES ALLOWED**

*Only the resources listed below are allowed in this exam.*

Any calculator which has the primary function of a calculator is allowed. For example, calculators on mobile phones or similar electronic devices are not allowed.

**DO NOT TAKE THIS PAPER FROM THE EXAM ROOM**

## Part A

**Question 1:** The spot price of an investment asset that provides no income is \$30 and the risk-free rate for all maturities (with continuous compounding) is 10%. What is the three-year forward price?

- \*A) \$40.50
- B) \$22.22
- C) \$33.00
- D) \$33.16

**Question 2:** Which of the following describes a call option?

- \*A) The right to buy an asset for a certain price
- B) The obligation to buy an asset for a certain price
- C) The right to sell an asset for a certain price
- D) The obligation to sell an asset for a certain price

**Question 3:** A limit order:

- A) Is an order to trade up to a certain number of futures contracts at a certain price
- \*B) Is an order that can be executed at a specified price or one more favorable to the investor
- C) Is an order that must be executed within a specified period of time
- D) None of the above

**Question 4:** Which of the following is true about a **long** forward contract?

- A) The contract becomes more valuable as the price of the underlying asset declines
- \*B) The contract becomes more valuable as the price of the underlying asset rises
- C) The contract is worth zero if the price of the underlying asset declines after the contract has been entered into.
- D) The contract is worth zero if the price of the underlying asset rises after the contract has been entered into

**Question 5:** A company can invest funds for five years at LIBOR plus 50 basis points. The five-year swap rate is 4%. What fixed rate of interest can the company earn by using the swap?

- A) 3.5%
- B) 4%
- \*C) 4.5%
- D) 5%

## Part B

**Question 1 (total of 10 marks):** A stock is expected to pay a 6-month dividend of \$0.25 per share for the foreseeable future. The current stock price is \$1.70 and the continuously compounded risk free rate is 6% p.a. for all maturities. An investor has just taken a short position in an 8-month futures contract on the stock. The last dividend payment was exactly 5 months ago. Therefore the next dividend of \$0.25 is in 1 month.

**Question 1a (4 marks):** Compute the futures price.

$$\begin{aligned} *F_{8\text{mth}} &= (S_0 - C_{1\text{mth}}/e^{(r*1/12)} - C_{7\text{mth}}/e^{(r*7/12)}) * e^{(r*8/12)} \\ &= (1.70 - 0.25/\exp(0.06*1/12) - 0.25/\exp(0.06*7/12)) * \exp(0.06*8/12) \\ &= (1.70 - 0.24875312 - 0.241401354) * 1.040810774 \\ &= 1.209845526 * 1.040810774 \\ &= 1.259220259 \end{aligned}$$

**Question 1b (2 marks):** Compute the initial value of the futures contract.

$$*V_0 = 0$$

**Question 1c (4 marks):** Six months later the price of the stock has risen to \$1.94 and the risk-free rate is unchanged.

Compute the new value of the **short** position in the futures contract. Note that the new value of the contract should be found, not the new futures price.

\*First find the expected stock price at  $t=8\text{mth}$ .

$$\begin{aligned} E(S_{8\text{mth}}) &= (S_{6\text{mth}} - C_{7\text{mth}}/\exp(r*1/12)) * \exp(r*2/12) \\ &= (1.94 - 0.25/\exp(0.06*1/12)) * \exp(0.06*2/12) \\ &= 1.708244194 \end{aligned}$$

Then find the current value of the long future which is the present value of  $S_t - K_t$

$$\begin{aligned} V_{6\text{mth},\text{long}} &= (E(S_{8\text{mth}}) - K_{8\text{mth}})/\exp(r*2/12) \\ &= (1.708244194 - 1.259220259)/\exp(0.06*2/12) = 0.444556072 \end{aligned}$$

$$V_{6\text{mth},\text{short}} = -V_{6\text{mth},\text{long}} = -0.444556072$$

**Question 2 (10 marks):** The below table summarises the borrowing costs confronting two companies:

Borrowing Costs		
	Fixed Rate	Floating Rate
Firm A	5.00%	6-month LIBOR + 0.9%
Firm B	5.80%	6-month LIBOR + 1.2%

Suppose Firm A wants to borrow at a floating rate and Firm B wishes to borrow fixed.

Design an intermediated swap that provides a bank with a spread of **20** basis points p.a., and divides the remaining swap benefits **equally** between the two companies.

Use a clearly labelled diagram to summarise the terms of the arrangement.

\*Firm A has an absolute advantage in both markets. But it has a comparative advantage in the fixed rate market so it should issue a fixed rate bond. Firm B has a comparative advantage in the floating rate market so it should issue a floating rate bond.

The total benefit available to all 3 parties including the bank is the absolute value of the difference of differences which is:

$$\text{TotalBenefitToABAndBank} = ||5.8-5| - |(0.9-1.2)|| = |0.8 - 0.3| = 0.5\%$$

Subtract the bank's spread top find the benefit to the banks:

$$\text{TotalBenefitToAandB} = 0.5\% - 0.2\% = 0.3\%$$

Firm A and B will share the benefits equally, so 0.15% (=0.3%/2) benefit each.

A receives $5-0.9+0.15=4.25\%$		B pays $5.8-1.2-0.15=4.45\%$	
A pays 5%	<b>Firm A</b>	<b>Bank</b>	<b>Firm B</b>
	A pays LIB	B receives LIB	B pays LIB+1.2%

**Question 3 (total of 10 marks):** Consider a 1 year **at**-the-money European call option on a **non-dividend** paying stock currently price at \$100. The risk free rate is **10%** pa continuously compounded and the standard deviation of the stock's continuously compounded returns is **22%** pa.

**Question 3a (3 marks):** Calculate  $d_1$ .

$$*d_1 = 0.564545455$$

**Question 3b (1 mark):** Calculate  $d_2$ .

$$*d_2 = 0.344545455$$

**Question 3c (2 mark):** Calculate  $N(d_1)$  using the tables in the back of this exam paper.

$$*N(d_1) = 0.713808513$$

**Question 3d (2 mark):** Calculate  $N(d_2)$  using the tables in the back of this exam paper.

$$*N(d_2) = 0.634781941$$

**Question 3e (2 marks):** Calculate the call option price.

$$*c = 13.94340603$$

**Question 4 (total of 10 marks):** Suppose a stock currently trades at \$32. A 6-month European call option with a strike price of \$30 has a premium of \$4.29, and a 6-month European put with the same strike price has a premium of \$2.64. Assume a 4% continuously compounded risk-free rate.

**Question 4a (4 marks):** What is the present value of expected dividends payable over the next 6 months, as implied by these premiums?

$$C_0 + K \cdot e^{-(r \cdot t)} = P_0 + S_0 - D_0$$

$$4.29 + 30/\exp(0.04 \cdot 6/12) = 2.64 + 32 - D_0$$

$$D_0 = 0.944$$

**Question 4b (6 marks):** Explain how you could exploit the situation if the company had no actual plans to pay a dividend. You must state the profit from your strategy as a present value. This problem is best presented by drawing an arbitrage table.

\*There are 3 ways to look at this question, either the put is over-priced, the stock is over-priced or the call is under-priced. Either way, all strategies should make a profit of \$0.944, the present value of the expected dividend which will not actually occur given your inside information.

One method: Buy the physical call since it's under-priced. Short the synthetic call (=short put, short stock and long bond) to balance out the risk (using puts and stocks) and make money straight away, requiring none of your own money (using bonds).

Viewing the below amounts as cash flows at time zero, then all positive cash flows are receipts to us now which are sell (short) transactions and all negative cash flows are payments from us now which are buy (long) transactions:

$$c_0 + K \cdot e^{-r \cdot T} = p_0 + (S_0 - D_0)$$

$$c_0 = p_0 + (S_0 - D_0) - K \cdot e^{-r \cdot T}$$

$$\text{ShortSyntheticCall} = \text{ShortPut} + \text{ShortStock} + \text{LongBond}$$

To find the amounts of these assets that we need to long and short to make a risk free and zero capital arbitrage, we'll use an arbitrage table:

Action	t=0	t=6mth, ST>K	t=6mth, ST<K
Long physical call	-4.29	ST-30	0
Short put	2.64	0	-(30-ST)
Short stock	32	-ST	-ST
Long bond	-29.406 (= - 30/exp(6/12*0.04)) (Step 4)	30 (Step 2)	30 (Step 2)
Total	0.944 (Step 5)	0 (Step 1)	0 (Step 1)

## Formulas

$$r_{\text{continuously compounded}} = \ln(1 + r_{\text{discrete}})$$

$$P_0 = \frac{P_t}{e^{t \cdot r_{\text{continuously compounded}}}}$$

$$r_{\text{discrete}} = e^{r_{\text{continuously compounded}}} - 1$$

$$P_0 = \frac{P_t}{(1 + r_{\text{discrete}})^t}$$

$$h^* = \rho_{S,F} \cdot \frac{\sigma_S}{\sigma_F}$$

$$N_{\text{no tailing}}^* = h^* \cdot \frac{Q_S}{Q_F}$$

$$N_{\text{tailing}}^* = h^* \cdot \frac{V_S}{V_F}$$

$$F = (S_0 - D_0) \cdot e^{r \cdot T}$$

$$f_{0, \text{long}} = (S_0 - D_0) - K \cdot e^{-r \cdot T}$$

$$f_{\text{long}} = -f_{\text{short}}$$

$$c_0 + K \cdot e^{-r \cdot T} = p_0 + (S_0 - D_0)$$

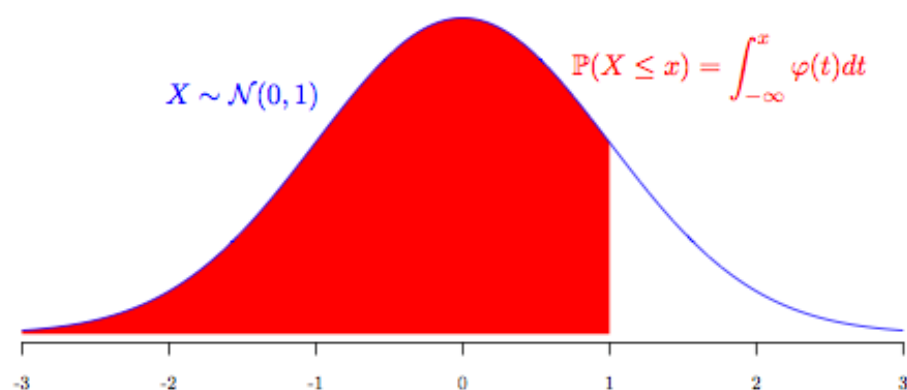
$$c_0 = S_0 \cdot N(d_1) - K \cdot e^{-r \cdot T} \cdot N(d_2)$$

$$p_0 = -S_0 \cdot N(-d_1) + K \cdot e^{-r \cdot T} \cdot N(-d_2)$$

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) \cdot T}{\sigma \cdot T^{0.5}}$$

$$d_2 = d_1 - \sigma \cdot T^{0.5} = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - \frac{\sigma^2}{2}\right) \cdot T}{\sigma \cdot T^{0.5}}$$





	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990