

Calculation Example: Mean and Median Prices

Question 1: A stock has an arithmetic average continuously compounded return (AALGDR) of **10%** pa, a standard deviation of continuously compounded returns (SDLGDR) of **16.45%** pa and current stock price of **\$1**.

In **20** years, what do you expect its **mean** and **median** prices to be?

Assume that stock prices are log-normally distributed.

Answer: The median price can be found with the formula.

$$MedianP_t = P_0 \cdot e^{AALGDR \cdot t}$$

$$MedianP_{20} = 1 \times e^{0.1 \times 20} = 7.389056099$$

The mean price is a little harder.

$$MeanP_t = P_0 \cdot e^{\left(AALGDR + \frac{SDLGDR^2}{2}\right) \cdot t}$$

$$MeanP_{20} = 1 \times e^{\left(0.1 + \frac{0.1645^2}{2}\right) \times 20} = 9.68523441$$

Calculation Example: Probabilities

Question 2: A stock has an arithmetic average continuously compounded return (AALGDR) of **10%** pa, a standard deviation of continuously compounded returns (SDLGDR) of **46.52%** pa and current stock price of **\$1**.

In **50** years, what do you expect its **mean** and **median** prices to be?

Assume that stock prices are log-normally distributed.

Answer: The median price can be found with the formula.

$$\text{Median}P_t = P_0 \cdot e^{AALGDR \cdot t}$$

$$\text{Median}P_{50} = 1 \times e^{0.1 \times 50} = 148.41$$

The mean price is a little harder.

$$MeanP_t = P_0 \cdot e^{\left(AALGDR + \frac{SDLGDR^2}{2}\right) \cdot t}$$

$$MeanP_{50} = 1 \times e^{\left(0.1 + \frac{0.4652^2}{2}\right) \times 50} = 33199.03$$

The mean is a lot bigger than the median.

Question 2: What is the probability of the share price exceeding the **median** price of \$148.41 in 50 years?

Answer: The median price is the 'middle price' when all possible prices are arranged from smallest to biggest. So the chance of achieving a price higher than the median is always 50%.

Question 3: What is the probability of the share price exceeding the **mean** price of \$33,199.03 in 50 years? Note that the mean is the arithmetic mean.

Answer: The chance of achieving a price more than the mean will be less than 50%.

To find the exact probability, convert the **log-normally** distributed prices into **normally** distributed continuously compounded returns (LGDR's).

$$\begin{aligned} LAAGDR_{50yr} &= \ln \left(\frac{MeanP_{50}}{P_0} \right) \\ &= \ln \left(\frac{33199.03}{1} \right) = 10.410276 \approx 1041\% \end{aligned}$$

$$AALGDR_{50yr} = AALGDR_{1yr} \times 50 = 0.1 \times 50 = 5 \approx 500\%$$

$$\begin{aligned} SDLGDR_{50yr} &= SDLGDR_{1yr} \times \sqrt{50} = 0.4652 \times \sqrt{50} \\ &= 3.289460746 \approx 329\% \end{aligned}$$

Note that these returns and the standard deviation are all measured over the 50 year period, they're not per annum.

By standardising these continuously compounded returns into Z scores we can then find the probability of exceeding the mean price.

$$\begin{aligned} Z &= \frac{x - \mu}{\sigma} \\ &= \frac{\ln(\text{Mean}P_{50}/1) - \ln(\text{Median}P_{50}/1)}{SDLGDR \times \sqrt{50}} \\ &= \frac{10.410276 - 5}{3.289460746} = 1.644730373 \end{aligned}$$

$$\text{Prob}(P_{50} < \text{Mean}P_{50}) = N(1.64473) = 0.95 = 95\%$$

$$\begin{aligned} \text{Prob}(P_{50} > \text{Mean}P_{50}) &= 1 - \text{Prob}(P_{50} < \text{Mean}P_{50}) \\ &= 1 - N(1.64473) = 1 - 0.95 = 5\% \end{aligned}$$

Therefore there's only a 5% chance of the stock price exceeding the mean expected future price of \$33,199.03 in 50 years!

However, there's a 50% chance of the stock price exceeding the median expected future price of \$148.41 in 50 years.

As you can see, the longer the time, the larger the difference between the mean and median, and the smaller the probability of exceeding the mean return.