## Annualised Percentage Rates (APR's)

Most interest rates are quoted as Annualised Percentage Rates (APR's). This is both by convention and in some countries by law. This is true for credit card rates, mortgage rates, bond yields, and many others.

The compounding period of an APR is not always explicitly stated. However, it can usually be assumed that the compounding frequency of an APR is the same as the payment frequency.

For example, a credit card might advertise an interest rate of $24 \%$ pa. This must be an APR since all advertised rates have to be APR's by law. Because credit cards are always paid off monthly, the compounding frequency is per month. Therefore
the interest rate is $24 \%$ pa given as an APR compounding monthly.

While APR's are the rate that you see quoted and advertised, unfortunately they cannot be used to find present or future values of cash flows. You must convert the APR to an effective rate before doing financial mathematics.

Confusion: APR's are sometimes called nominal rates. Unfortunately, nominal has another meaning related to inflation (nominal versus real returns). We will avoid calling APR's nominal rates in these notes.

## Effective Rates

Effective rates compound only once over their time period, and the time period can be of any length, not necessarily annual.

Effective rates can be used to discount cash flows.
APR's cannot be used to discount cash flows, they must be converted to effective rates first.

Note that all of the calculation examples up to here have assumed that the interest rate given is an effective rate.

## Calculation Example: Present Values and Effective Rates

Question: What is the present value of receiving \$100 in one year when the effective monthly rate is $1 \%$ ?
Answer: Since the effective interest rate is per month, the time period must also be in months, so:
$V_{o}=\frac{C_{t}}{(1+r)^{t}}$

$$
\begin{aligned}
& =\frac{100}{(1+0.01)^{12}} \\
& =88.7449
\end{aligned}
$$

## APR's and Effective Rates

- An APR compounding monthly is equal to 12 multiplied by the effective monthly rate.

$$
r_{A P R ~ c o m p ~ m o n t h l y ~}=r_{\text {eff monthly }} \times 12
$$

- An APR compounding semi-annually is equal to 2 multiplied by the effective 6 month rate.
$r_{A P R \text { comp per } 6 m t h s}=r_{\text {eff } 6 m t h} \times 2$
- An APR compounding daily is equal to 365 multiplied by the effective daily rate.

```
rAPR comp daily }=\mp@subsup{r}{\mathrm{ eff daily }}{}\times36
```


## Example: Future Values with APR's

Question: How much will your credit card debt be in 1 year if it's $\$ 1,000$ now and the interest rate is $24 \% \mathrm{pa}$ ?
Wrong Answer: $V_{t}=C_{0}(1+r)^{t}=1000(1+0.24)^{1}=1,240$
Answer: Since credit cards are paid off per month and rates are by default given as APR's, the $24 \%$ must be an APR compounding monthly. Therefore the effective monthly rate will be the APR divided by 12 .

$$
\begin{aligned}
& r_{\text {eff monthly }}=\frac{r_{A P R \text { comp monthly }}}{12}=\frac{0.24}{12}=0.02 \\
& \begin{array}{l}
V_{12 \text { mths }}=C_{0}\left(1+r_{\text {eff monthly }}\right)^{t_{\text {months }}} \\
\quad=1000(1+0.02)^{12}=1,268.2418
\end{array}
\end{aligned}
$$

## Converting Effective Rates To Different Time Periods

Compounding the rate higher (up to a longer time period):
$r_{\text {eff annual }}=\left(1+r_{\text {eff monthly }}\right)^{12}-1$
$r_{\text {eff semi-annual }}=\left(1+r_{\text {eff monthly }}\right)^{6}-1$
$r_{\text {eff quarterly }}=\left(1+r_{\text {eff monthly }}\right)^{3}-1$
Compounding the rate lower (down to a shorter time period):
$r_{\text {eff monthly }}=\left(1+r_{\text {eff annual }}\right)^{\frac{1}{12}}-1$
$r_{\text {eff daily }}=\left(1+r_{\text {eff annual }}\right)^{\frac{1}{365}}-1$

## Calculation Example: Converting Effective Rates

Question: A stock was bought for $\$ 10$ and sold for $\$ 15$ after 7 months. No dividends were paid. What was the effective annual rate of return?

## Answer:

First we find the return over 7 months. This will be the effective 7 month rate of return. Note that the time period is in 7-month blocks:
$V_{0}=\frac{V_{t}}{(1+r)^{t}}$

$$
V_{0}=\frac{V_{7 \text { months }}}{\left(1+r_{\text {eff 7month }}\right)^{1_{\text {seven month period }}}}
$$

$$
10=\frac{15}{\left(1+r_{\text {eff } 7 \text { month }}\right)^{1}}
$$

$\left(1+r_{\text {eff } 7 \text { month }}\right)^{1}=\frac{15}{10}$
$r_{\text {eff } 7 \text { month }}=\frac{15}{10}-1=0.5=50 \%$, which is the effective 7 month rate.

Now we need to convert the effective 7 month rate to an effective annual rate (EAR). This can be done by 'compounding up' by $12 / 7$ in one step:

$$
\begin{aligned}
r_{e f f \text { annual }} & =\left(1+r_{\text {eff } 7 m t h}\right)^{\frac{12}{7}}-1 \\
& =(1+0.5)^{12 / 7}-1=1.0039=100.39 \%
\end{aligned}
$$

Or it can be broken down into steps:

- Compounding the 7-month rate down to a monthly rate:

$$
\begin{aligned}
r_{\text {eff monthly }} & =\left(1+r_{\text {eff } 7 m t h}\right)^{1 / 7}-1 \\
& =(1+0.5)^{1 / 7}-1=0.059634=5.9634 \%
\end{aligned}
$$

- Then compound the monthly rate up to a 12 -month (annual) rate:

$$
\begin{aligned}
r_{e f f \text { annual }} & =\left(1+r_{\text {eff,monthly }}\right)^{12}-1 \\
& =(1+0.059634)^{12}-1=1.0039=100.39 \%
\end{aligned}
$$

## Calculation Example: Converting APR's to Effective Rates

Question: You owe a lot of money on your credit card. Your credit card charges you $9.8 \%$ pa, given as an APR compounding per month.

You have the cash to pay it off, but your friend wants to borrow money from you and offers to pay you an interest rate of $10 \%$ pa given as an effective annual rate. Assume that your friend will definitely pay you back (no credit risk).

Should you use your cash to pay off your credit card or lend it to your friend?

Answer: The loan's 10\% effective annual rate can't be immediately compared to the credit card's 9.8\% APR compounding per month.

Method 1: Convert the credit card's 9.8\% APR compounding per month to an effective annual rate:

$$
\begin{aligned}
r_{\text {eff monthly }} & =\frac{r_{A P R \text { comp monthly }}}{12}=\frac{0.098}{12}=0.0081667 \\
r_{\text {eff annual }} & =\left(1+r_{\text {eff monthly }}\right)^{12}-1 \\
& =(1+0.0081667)^{12}-1=0.1025=10.25 \%
\end{aligned}
$$

So the credit card's 9.8\% APR compounding per month converts to an effective annual rate of $10.25 \%$. This is more than the loan's 10\% effective annual rate.

You should pay off your credit card which costs $10.25 \%$ rather than lend to your friend which only earns $10 \%$, where both rates are effective annual rates.

Method 2: Convert the loan's 10\% effective annual rate to an APR compounding per month:

$$
\begin{aligned}
& r_{\text {eff monthly }}=\left(1+r_{\text {eff annual }}\right)^{1 / 12}-1 \\
& =(1+0.1)^{1 / 12}-1=0.00797414043 \\
& r_{\text {APR comp monthly }}=r_{\text {eff monthly }} \times 12 \\
& =0.00797414043 \times 12 \\
& =0.09568968514=9.568968514 \% p a
\end{aligned}
$$

So the $10 \%$ effective annual rate that you can lend at converts to a $9.569 \%$ APR compounding per month. This is less than your $9.8 \%$ pa cost of funds using your credit card, where both are APR's compounding monthly, so don't lend to your friend.

## Calculation Examples

Question: Assume 30 days per month and 360 days per year.
Convert a $\mathbf{9 . 8} \%$ APR compounding per month into the following:

$$
\begin{aligned}
& r_{\text {eff } 6 \text { month }}=\left(1+\frac{0.098}{12}\right)^{6}-1=0.050011377 \text { per } 6 \text { months } \\
& r_{A P R ~ c o m p ~ 6 m o n t h s ~}=\left(\left(1+\frac{0.098}{12}\right)^{6}-1\right) \times 2=0.100022754 \mathrm{pa}
\end{aligned}
$$

$$
r_{e f f} \text { daily }=\left(1+\frac{0.098}{12}\right)^{\frac{1}{30}}-1=0.000271153 \text { per day }
$$

$$
r_{A P R ~ c o m p ~ d a i l y}=\left(\left(1+\frac{0.098}{12}\right)^{\frac{1}{30}}-1\right) \times 360=0.097615231 \mathrm{pa}
$$

## Calculation Examples

Question: Assume 30 days per month and 360 days per year. Convert a 10\% effective annual rate ( $r_{\text {eff annual }}$ ) into the following:

$$
\begin{aligned}
& r_{\text {eff } 6 \text { month }}=(1+0.1)^{6 / 12}-1=0.048808848 \text { per } 6 \text { months } \\
& r_{A P R ~ c o m p ~ 6 m o n t h s ~}=\left((1+0.1)^{6 / 12}-1\right) \times 2=0.097617696 \text { pa }
\end{aligned}
$$

$$
r_{e f f ~ d a i l y}=(1+0.1)^{1 / 360}-1=0.000264786 \text { per day }
$$

$$
r_{A P R} \text { comp daily }=\left((1+0.1)^{1 / 360}-1\right) \times 360=0.095322798 \text { pa }
$$

$$
r_{e f f} 2 \text { year }=(1+0.1)^{2}-1=0.21 \text { per } 2 \text { years }
$$

$$
r_{e f f ~}^{1 \text { second }}=(1+0.1)^{1 /(12 \times 30 \times 24 \times 60 \times 60)}-1=0.000000003064 / \mathrm{s}
$$

$$
r_{A P R} \text { comp second }=r_{\text {eff } 1 \text { second }} \times 12 \times 30 \times 24 \times 60 \times 60
$$

$$
=0.0953101823 p a \approx \ln (1+0.1)=r_{\text {continuously compounded } p a}
$$

## Questions: APR's and Effective Rates

http://www.fightfinance.com/?q=290,330,16,26,131,49,64,26 5,

