***Bond Pricing in between coupons***

To price a bond in between coupon periods at time *t*, grow the bond price $P\_{0}$ forward by the yield to maturity:

$P\_{t} =P\_{0}\left(1+r\_{eff}\right)^{t}$ where:

$P\_{t}$ is the bond price at the current time *t* and 0<*t*<1;

$C\_{1}$ is the next coupon payment at time one;

*T* is the number of coupons remaining to be paid;

$r\_{eff}$ is the yield to maturity as an effective rate per coupon period;

$P\_{0}$ is the bond price one period before the next coupon $C\_{1}$;

$$P\_{0} =\frac{C\_{1}}{r\_{eff}}\left(1-\frac{1}{\left(1+r\_{eff}\right)^{T}}\right)+\frac{F\_{T}}{\left(1+r\_{eff}\right)^{T}}$$

***Calculation Example: Bond pricing in between coupons***

**Question:** A **3** year government bond paying **10**% pa semi-annual coupons with a face value of $**100** was issued **4** months ago at a yield of **5**% pa. Find the current price of the bond.

Ignore the actual number of days in each month and assume that every month is 1/12 of a year.

**Answer:** Let the next coupon payment in 6 months be time 1. Let’s find the bond price one (semi-annual) coupon period before, which is time zero, with 6 semi-annual coupons left:

$$P\_{0}=\frac{0.1×100/2}{0.05/2}\left(1-\frac{1}{\left(1+0.05/2\right)^{3×2}}\right)+\frac{100}{\left(1+0.05/2\right)^{3×2}}$$

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$$ =\frac{5}{0.025}\left(1-\frac{1}{\left(1+0.025\right)^{6}}\right)+\frac{100}{\left(1+0.025\right)^{6}}$$

$$ =113.770313404$$

Now grow the bond price that extra 4 months forward, which is 4/6 (=0.66667) semi-annual periods, to get to the current time:

$$P\_{4months}=P\_{0}\left(1+r\_{APR comp 6 months}/2\right)^{4/6}$$

$$ =113.770313404×\left(1+0.05/2\right)^{4/6}$$

$$ =115.6586711$$

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***Calculation Example: Bond pricing in between coupons***

**Question:** A **10** year government bond paying **3**% pa semi-annual coupons with a face value of $**100** was issued 8 months ago on 15 December 2021 at a yield of **3**% pa.

Today is 15 August 2022 and yields are now **2.8**% pa. What is the current price of the bond?

Ignore the actual number of days in each month and assume that every month is 1/12 of a year, so the bond was issued 8 months ago from today, 15 August 2022.

**Answer**: Let the issue date 15 December 2021 be time zero.There are only 19 semi-annual coupons left, since the first was already paid on 15 June 2022. The bond’s next coupon ($C\_{2}$) will be paid on 15 December 2022. The bond price one period before coupon $C\_{2}$ is:

$$P\_{1}=\frac{C\_{2}}{r\_{eff}}\left(1-\frac{1}{\left(1+r\_{eff}\right)^{19}}\right)+\frac{F\_{19}}{\left(1+r\_{eff}\right)^{19}}=P\_{\begin{array}{c}15Jun2022,\\6 months \\after issue\end{array}}$$

$$ =\frac{0.03×100/2}{0.028/2}\left(1-\frac{1}{\left(1+0.028/2\right)^{19}}\right)+\frac{100}{\left(1+0.028/2\right)^{19}}$$

$$ =24.87274706+76.78543608=101.6581831$$

$$P\_{1.3333}=P\_{1}\left(1+0.028/2\right)^{0.3333}=P\_{\begin{array}{c}15Aug2022,\\8 months \\after issue\end{array}}$$

$$ =101.6581831×\left(1+0.028/2\right)^{2/6}=102.1303912$$

The subscripts are in coupon periods so they correspond to the graph and exponents shown in the formula. So for example:

* $P\_{0}$ is the initial price when the bond was issued on 15 December 2021.
* $P\_{1}$ is the price 1 semi-annual period (6 months) after the bond was issued and corresponds to 15 June 2022.
* $P\_{1.3333}$ is the price 1.3333 semi-annual period (8 months) after the bond was issued and corresponds to 15 August 2022. It’s the current time that we’re trying to price the bond.
* $C\_{2}$ is the coupon 2 semi-annual periods (1 year) after the bond was issued and corresponds to 15 December 2022.

The graph helps visualize the problem.

