Bond Pricing in between coupons

To price a bond in between coupon periods at time t, grow the bond price P_0 forward by the yield to maturity:

$P_t = P_0\left(\mathbf{1} + \boldsymbol{r_{eff}}\right)^{\boldsymbol{t}}$ where:

P_t is the bond price at the current time t and $0<t<1$;

C_1 is the next coupon payment at time one;

T is the number of coupons remaining to be paid;

r_{eff} is the yield to maturity as an effective rate per coupon period;

P_0 is the bond price one period before the next coupon C_1;

$$P_0 = \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) + \frac{F_T}{\left(1 + r_{eff}\right)^T}$$

Calculation Example: Bond pricing in between coupons

Question: A **3** year government bond paying **10**% pa semi-annual coupons with a face value of $**100** was issued **4** months ago at a yield of **5**% pa. Find the current price of the bond.

Ignore the actual number of days in each month and assume that every month is 1/12 of a year.

Answer: Let the next coupon payment in 6 months be time 1. Let's find the bond price one (semi-annual) coupon period before, which is time zero, with 6 semi-annual coupons left:

$$P_0 = \frac{0.1 \times 100/2}{0.05/2}\left(1 - \frac{1}{(1+0.05/2)^{3\times 2}}\right) + \frac{100}{(1+0.05/2)^{3\times 2}}$$

$$P_0 = \frac{0.1 \times 100/2}{0.05/2}\left(1 - \frac{1}{(1+0.05/2)^{3\times 2}}\right) + \frac{100}{(1+0.05/2)^{3\times 2}}$$

$$= \frac{5}{0.025}\left(1 - \frac{1}{(1+0.025)^{6}}\right) + \frac{100}{(1+0.025)^{6}}$$

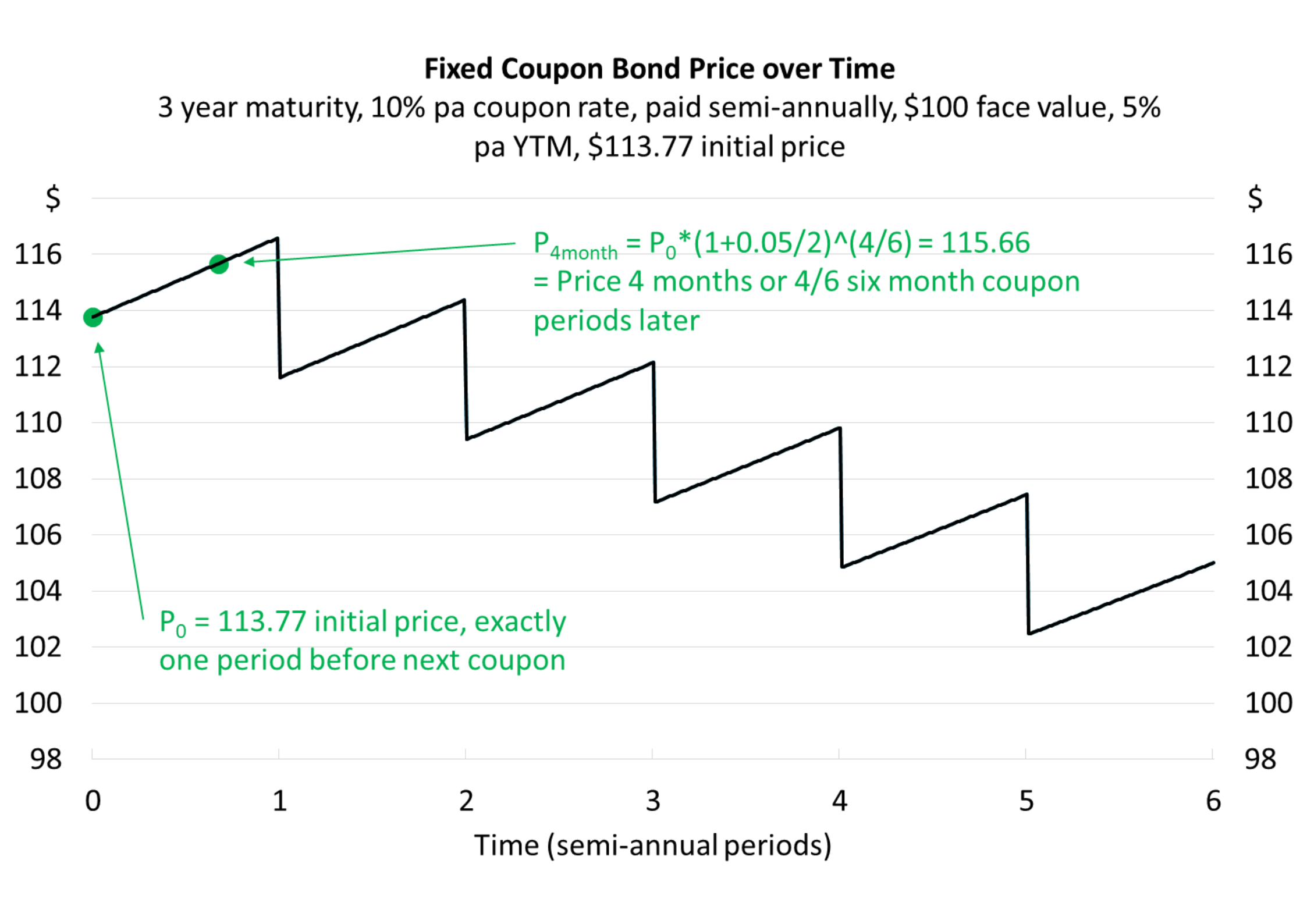
$$= 113.770313404$$

Now grow the bond price that extra 4 months forward, which is 4/6 (=0.66667) semi-annual periods, to get to the current time:

$$P_{4months} = P_0\left(1 + r_{APR\ comp\ 6\ months}/2\right)^{4/6}$$

$$= 113.770313404 \times (1+0.05/2)^{4/6}$$

$$= 115.6586711$$


Fixed Coupon Bond Price over Time
3 year maturity, 10% pa coupon rate, paid semi-annually, $100 face value, 5% pa YTM, $113.77 initial price
$
116
114
112
110
108
106
104
102
100
98
$
116
114
112
110
108
106
104
102
100
98
P4month = P0*(1+0.05/2)^(4/6) = 115.66
= Price 4 months or 4/6 six month coupon periods later
P0 = 113.77 initial price, exactly one period before next coupon
0
1
2
3
4
5
6
Time (semi-annual periods)

Calculation Example: Bond pricing in between coupons

Question: A **10** year government bond paying **3**% pa semi-annual coupons with a face value of $**100** was issued 8 months ago on 15 December 2021 at a yield of **3**% pa.

Today is 15 August 2022 and yields are now **2.8**% pa. What is the current price of the bond?

Ignore the actual number of days in each month and assume that every month is 1/12 of a year, so the bond was issued 8 months ago from today, 15 August 2022.

Answer: Let the issue date 15 December 2021 be time zero. There are only 19 semi-annual coupons left, since the first was already paid on 15 June 2022. The bond's next coupon (C_2) will be paid on 15 December 2022. The bond price one period before coupon C_2 is:

$$P_1 = \frac{C_2}{r_{eff}}\left(1 - \frac{1}{(1+r_{eff})^{19}}\right) + \frac{F_{19}}{(1+r_{eff})^{19}} = P_{\substack{15Jun2022,\\ 6\,months\\ after\ issue}}$$

$$= \frac{0.03 \times 100/2}{0.028/2}\left(1 - \frac{1}{(1+0.028/2)^{19}}\right) + \frac{100}{(1+0.028/2)^{19}}$$

$$= 24.87274706 + 76.78543608 = 101.6581831$$

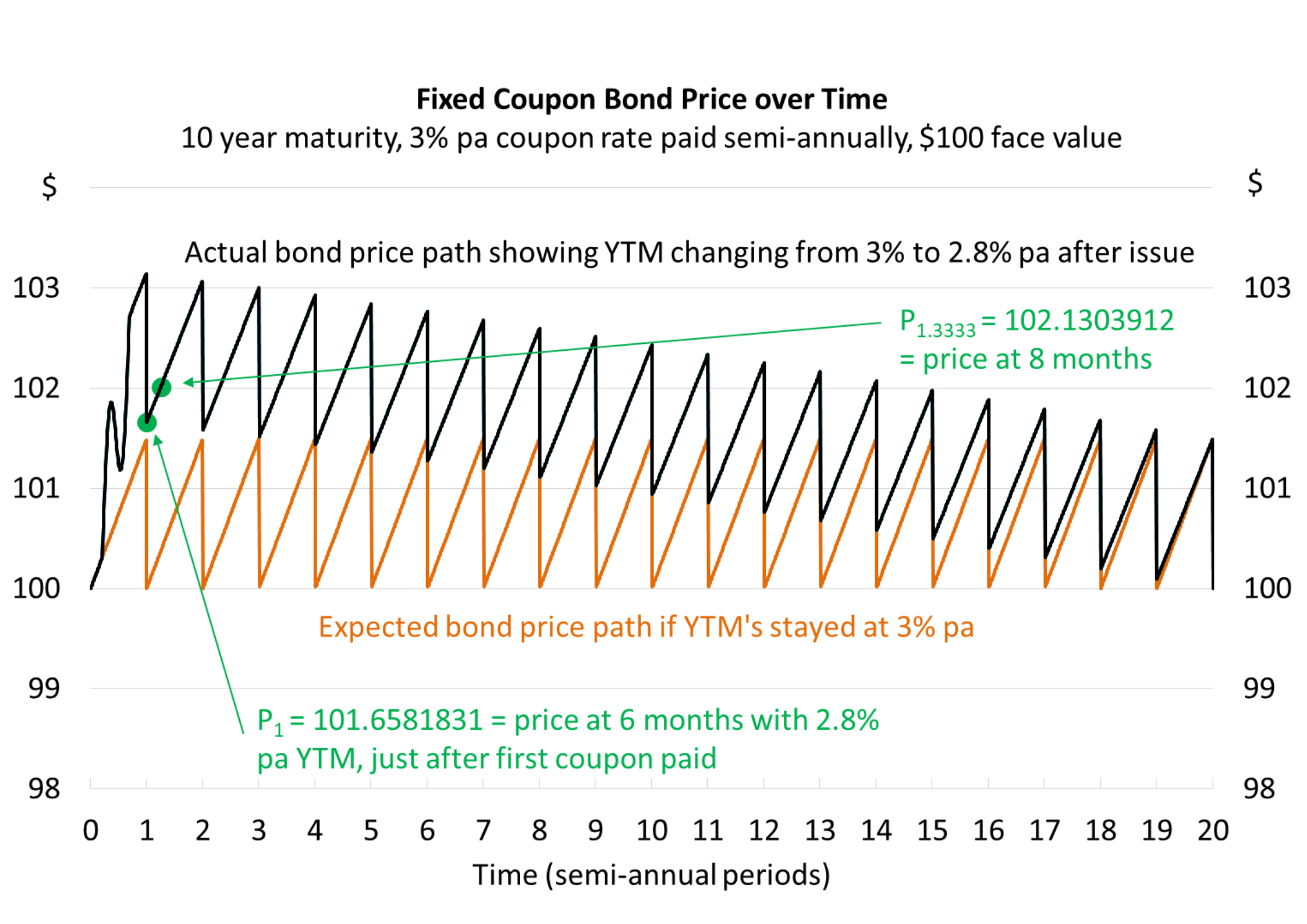
$$P_{1.3333} = P_1(1+0.028/2)^{0.3333} = P_{\substack{15Aug2022,\\ 8\,months\\ after\ issue}}$$

$$= 101.6581831 \times (1+0.028/2)^{2/6} = 102.1303912$$

The subscripts are in coupon periods so they correspond to the graph and exponents shown in the formula. So for example:

- P_0 is the initial price when the bond was issued on 15 December 2021.
- P_1 is the price 1 semi-annual period (6 months) after the bond was issued and corresponds to 15 June 2022.
- $P_{1.3333}$ is the price 1.3333 semi-annual period (8 months) after the bond was issued and corresponds to 15 August 2022. It's the current time that we're trying to price the bond.
- C_2 is the coupon 2 semi-annual periods (1 year) after the bond was issued and corresponds to 15 December 2022.

The graph helps visualize the problem.


Fixed Coupon Bond Price over Time
10 year maturity, 3% pa coupon rate paid semi-annually, $100 face value
$
$
Actual bond price path showing YTM changing from 3% to 2.8% pa after issue
103
102
101
100
99
98
$P_{1.3333}$ = 102.1303912
= price at 8 months
Expected bond price path if YTM's stayed at 3% pa
P_1 = 101.6581831 = price at 6 months with 2.8%
pa YTM, just after first coupon paid
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Time (semi-annual periods)