#### Valuation with Debt

If a firm has no debt, we say it is 'unlevered' or 'all-equity'. Let:

- *V<sub>U</sub>* be the **un**levered asset value;
- $V_L$  the levered asset value;
- $E_U$  the **un**levered equity value;

 $E_{LwITS}$  the levered equity value with interest tax shields;

 $E_{LxITS}$  the levered equity value excluding interest tax shields;

 $V_{ITS}$  the present value of interest tax shields.

Some formulas that must hold:

 $V_U = E_U$ , since debt is zero in an unlevered firm.

 $V_L = V_{II} + V_{ITS}$ , since the value of the levered firm is equal to that of the unlevered firm, plus the interest tax shields.  $E_{LWITS} = E_{LXITS} + V_{ITS}$  $V_L = D + E_{LWITS}$  $V_L = D + E_{L_{XITS}} + V_{ITS}$ Define profits and cash flows as:  $NI = (Rev - COGS - FC - Depr - IntExp).(1 - t_c) = NPAT$  $FFCF = NI + Depr - CapEx - \Delta NOWC + IntExp = CFFA$ 

 $NOPAT = (Rev - COGS - FC - Depr - \mathbf{0}).(1 - t_c)$  $OFCF = NOPAT + Depr - CapEx - \Delta NOWC + \mathbf{0}$ 

These equations give us three different methods to find the value of a levered firm  $(V_L)$ .

# Valuation with Interest Tax Shields in the Discount Rate: 'Textbook Method'

The most commonly used method to value a levered project (find  $V_L$ ) is to calculate its cash flows as if it is all-equity financed, so there is no debt and no interest expense. That is, find the operating free cash flows *OFCF* s.

Then discount the OFCFs using the firm's after tax weighted average cost of capital,  $WACC_{after-tax}$ .

This will give the correct value of the firm including the present value of the interest tax shields.

This method takes the interest tax shield into account in the **discount rate** rather than the **cash flow**.

There are two key assumptions:

- the firm has a target debt-to-assets ratio (D/V) or debt-toequity ratio (D/E) that it sticks to, and
- the project is of similar risk to the rest of the business.

It might seem curious that the tax shield benefit is included in the project's value when we used the OFCF which pretends debt there's no interest expense on debt.

But the interest tax shield benefit is included in the discount rate (the after-tax WACC) rather than the cash flow (the OFCF). The cost of debt in the after-tax WACC,  $r_D$ , is being multiplied by  $(1 - t_c)$  which reduces the cost of debt by the amount of the tax shield. The lower discount rate (WACC) makes for a higher value (V), and this increase in value is due to the interest tax shields.

## **Calculation Example: Firm Valuation**

**Question 1**: A firm has a target debt-to-assets ratio  $\left(\frac{D}{V_{T}}\right)$  of

25%.

- The cost of levered equity (with the interest tax shield) is 10%. The cost of debt is 5%.
- The tax rate is 30%.

The firm's operating free cash flows (OFCFs) are \$10m each year which is expected to be earned at the end of every year forever.

Find the value of the levered firm  $(V_L)$ .

**Answer:** We are already given the OFCF, so the next step is to find the after-tax WACC:

$$WACC_{after-tax} = r_d. (1 - t_c). \frac{D}{V} + r_e. \frac{E}{V}$$
  
= 0.05 × (1 - 0.3) × 0.25 + 0.1 × 0.75  
= 0.08375

Since the positive cash flows go on forever, we will use the perpetuity formula  $(V_0 = \frac{C_1}{r-g})$  to find the value of the levered firm:

 $V_L = PV[OFCF, discounted by WACC_{after-tax}]$  $V_L = \frac{10m}{0.08375} = \$119.403m$ 

# Valuation with Interest Tax Shields in the Cash Flow

The second (harder) way to value a project is to calculate its firm free cash flows (FFCF, also known as cash flow from assets CFFA), and then to discount these using the WACC before tax.

This method takes the interest tax shield into account in the cash flow rather than the discount rate. This method also assumes that:

- the firm has a target debt-to-assets ratio (D/V) or debt-toequity ratio (D/E) that it sticks to, and
- the project is of similar risk to the rest of the business.

## **Calculation Example: Firm Valuation**

In the previous question the firm had a levered value ( $V_L$ ) of \$119.403m.

**Question 2**: Show that the levered value of the firm using the harder method gives the same result.

**Answer:** 

$$WACC_{before-tax} = r_D \cdot \frac{D}{V} + r_E \cdot \frac{E}{V}$$
$$= 0.05 \times 0.25 + 0.1 \times 0.75$$
$$= 0.0875$$

We need to find the FFCF. This depends on the yearly interest tax shield which depends the amount of debt D. Since the debt-to-assets ratio is 25%:

 $\frac{D}{V_L} = 0.25$  $D = 0.25 \times V_L$ Let's substitute this into the below equation,  $FCFF = OFCF + IntExp \times t_c$  $= OFCF + D \times r_d \times t_c$  $= OFCF + 0.25 \times V_L \times r_d \times t_c$  $= 10m + 0.25 \times V_L \times 0.05 \times 0.3$ 

Since the positive cash flows go on forever, we will use the perpetuity formula  $(V_0 = \frac{C_1}{r-g})$  to find the value of the levered firm:

 $V_L = PV[FFCF, discounted by WACC_{before-tax}]$  $V_L = \frac{10m + 0.25 \times V_L \times 0.05 \times 0.3}{0.0875}$  $V_L \times 0.0875 - 0.25 \times V_L \times 0.05 \times 0.3 = 10m$  $V_L \times (0.0875 - 0.25 \times 0.05 \times 0.3) = 10m$  $V_L = \frac{10m}{(0.0875 - 0.25 \times 0.05 \times 0.3)}$ = \$119.4030*m* Which is the same as before!

# Valuation with Interest Tax Shields Added Separately: APV Method

The Adjusted Present Value (APV) method is to discount the OFCF by the before-tax WACC (which is the same as the unlevered cost of equity) to get the unlevered value of assets ( $V_U$ , which is equal to  $E_U$ ), and then add the present value of interest tax shields on separately ( $V_{ITS}$ ), so

 $V_L = V_U + V_{ITS}$ 

Where:

 $V_U = PV[OFCF, discounted by WACC_{before-tax}]$ 

## Firm and Project Valuation

The main problems to avoid are:

- Double-counting the interest tax shield in both the discount rate and the cash flow. It should only be included in one of them. Double-counting will lead to valuations that are too high.
- Not including the tax shield at all, in the discount rate or in the cash flow. This will lead to valuations that are too low.

#### **Theory Examples: Valuation**

**Question 1**: A levered project's FFCF is calculated. It is discounted using the after-tax WACC. Is this correct?

**Answer**: No, this will double-count the interest tax shield. Interest is included in the cash flow (FFCF that includes interest expense), so the tax shield per year will be *IntExp*. *t<sub>c</sub>*.

The discount rate is the after-tax WACC which also accounts for the tax shield since it reduced the cost of debt by  $(1 - t_c)$ . So the tax shield will be included in the cash flow and the discount rate which is wrong.

The asset value price will be too big.

**Question 2**: A levered project's OFCF is discounted using the after-tax WACC. Is this correct?

**Answer**: Yes, this is the textbook method. The tax shield is only included in the discount rate. So long as the project has the same risk as the firm, and the firm keeps to its debt to equity ratio, then the project will be correctly valued.

**Question 3**: A levered project's OFCF is discounted using the pre-tax WACC. Is this correct?

**Answer**: No, tax shields are not included at all. This calculation will give the unlevered value of the project  $V_U$  which will be too low compared to the correct value of the project's assets  $V_L$ .

**Question 4**: A levered project's FFCF is discounted using the pre-tax WACC. Is this correct?

**Answer**: Yes, this is the harder method. The tax shield is included in the cash flow, and not the discount rate so this will give the correct, levered value of the project.

Another difficulty is that the WACC assumes a constant debtto-equity ratio. This means that when the value of the firm changes, for example, after a dividend is paid, then the amount of debt and also the interest tax shield needs to be recalculated. This can be very laborious. **Question 5**: A levered project's FFCF is discounted using the pre-tax WACC to find the value of the unlevered firm. Then the present value of tax shields are added separately. Is this correct?

**Answer**: No, this double-counts the tax shields since the value of the firm is calculated using the 'harder method', and then the tax shields are added again.

**Question 6**: A levered project's OFCF is discounted using the pre-tax WACC to find the value of the unlevered project. Then the present value of tax shields are added separately. Is this correct?

**Answer**: Yes, this is the APV method of calculating the unlevered value of the firm and adding on the present value of tax shields separately.

# Questions: Interest tax shield cash flows and levered business valuation

http://www.fightfinance.com/?q=67,206,223,296,68,89,113,2 38,273,367,368,369,371,375,413,370,