

Calculation Example

Q1) Find the discrete yearly returns of stocks CBA and BHP from the following price data.

Date	Adjusted Closing Price (\$)	
	CBA	BHP
1/1/2007	48.39	25.17
2/1/2008	47.77	36.31
2/1/2009	26.01	30.11
4/1/2010	51.47	38.9
4/1/2011	52.46	44.25

To find CBA's return from 2010 to 2011,

$$r_{CBA, 2010 \rightarrow 2011} = \frac{p_{4/1/2011}}{p_{4/1/2010}} - 1$$
$$= \frac{52.46}{51.47} - 1 = 0.0192$$

And so on. Here are the complete results:

Date	Return (p.a.)	
	CBA	BHP
1/1/2007		
2/1/2008	-0.0128	0.4426
2/1/2009	-0.4555	-0.1708
4/1/2010	0.9789	0.2919
4/1/2011	0.0192	0.1375

Q2) Calculate the arithmetic mean, variance and standard deviation of returns.

$$\bar{r}_{CBA, \substack{2007 \rightarrow \\ 2011, pa}} = \frac{r_{CBA,07 \rightarrow 08} + r_{CBA,08 \rightarrow 09} + r_{CBA,09 \rightarrow 10} + r_{CBA,10 \rightarrow 11}}{n}$$

$$= \frac{-0.0128 - 0.4555 + 0.9789 + 0.0192}{4} = 0.1324$$

$$var(r) = \sigma^2 = \frac{\sum_{i=1}^n [(r_i - \bar{r})^2]}{n-1}$$

$$\sigma^2_{CBA, 2007 \rightarrow 2011, p.a.} = \frac{\left[(-0.0128 - 0.1324)^2 + (-0.4555 - 0.1324)^2 + (0.9789 - 0.1324)^2 + (0.0192 - 0.1324)^2 \right]}{4-1} = 0.3653$$

$$\sigma_{CBA, 2007 \rightarrow 2011, p.a.}^2 = 0.3653$$

$$\begin{aligned}\sigma_{CBA, 2007 \rightarrow 2011, p.a.} &= \sqrt{0.3653} \\ &= 0.6044\end{aligned}$$

Similarly for BHP. Here are the complete results:

	CBA	BHP
Return	0.1324	0.1753
Variance	0.3653	0.0687
St. dev.	0.6044	0.2622

Q3) Calculate the covariance and correlation of their returns.

$$cov(r_A, r_B) = \sigma_{A,B} = \frac{\sum_{i=1}^n [(r_{A,i} - \bar{r}_A)(r_{B,i} - \bar{r}_B)]}{n - 1}$$

$$\sigma_{CBA,BHP,2007 \rightarrow 2011, p.a.} =$$

$$\frac{[(-0.0128 - 0.1324) \times (0.4426 - 0.1753) +] \\ [(-0.4555 - 0.1324) \times (-0.1708 - 0.1753) +] \\ [(0.9789 - 0.1324) \times (0.2919 - 0.1753) +] \\ [(0.0192 - 0.1324) \times (0.1375 - 0.1753) +]}{4 - 1}$$

$$\sigma_{CBA,BHP,2007 \rightarrow 2011, p.a.} = 0.0892$$

$$correl(r_1, r_2) = \rho_{1,2} = \frac{cov(r_1, r_2)}{sd(r_1).sd(r_2)} = \frac{\sigma_{1,2}}{\sigma_1.\sigma_2}$$

$$\rho_{CBA,BHP,2007-2011,p.a.} = \frac{0.0892}{0.6044 \times 0.2622} \\ = 0.5629$$

	CBA	BHP
Covariance		0.0892
Correlation		0.5629

In conclusion, BHP has a higher return and lower risk than CBA. The correlation is not near one so a fair amount of diversification is possible.