***Portfolios and Diversification***

Let P be a $100 portfolio with $50 invested in A and $50 in B.

* Therefore P is an equal-weighted portfolio in A and B. So:

$x\_{A}=0.5$, $x\_{B}=0.5$

* Return will be exactly halfway at 0.15.
* Standard deviation is likely to be less than halfway (0.2) due to diversification - we didn’t “put all of our eggs in one basket”.
* So P will be somewhere along the dashed red line. The standard deviation of P depends on the **correlation** between A and B.

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***How Correlation Affects Diversification***

* ******$ρ\_{A,B}=-1$, lots of diversification since A and B move in opposite directions.
* $ρ\_{A,B}=0$, diversification since A and B move independently and will sometimes cancel each other out.
* $ρ\_{A,B}=1$, no diversification at all since A and B move with each other in the same ratio. In this case $σ\_{P}=0.2$.

***Portfolio Return and Variance***

Portfolio return for $n$ stocks with weights $x$:

$$r\_{P}=x\_{1}.r\_{1}+x\_{2}.r\_{2}+…+x\_{n}.r\_{n}=\sum\_{i=1}^{n}(x\_{i}.r\_{i})$$

Note that the weights must sum to one:

$$x\_{1}+x\_{2}+…+x\_{n}=1$$

Portfolio variance for **2** stocks with weights $x\_{1}$ and $x\_{2}$:

$$σ\_{P}^{2}=x\_{1}^{2} .σ\_{1}^{2}+x\_{2}^{2} .σ\_{2}^{2}+2.x\_{1}.x\_{2}.σ\_{1,2}$$

***Portfolio Return and Variance Example***

**Question:** Portfolio P in the diagram has a weight of 0.5 in A and 0.5 in B. The correlation between A and B is 0.05. Find the return and variance of portfolio P.

**Answer:**

$$r\_{P}=x\_{A}.r\_{A}+x\_{B}.r\_{B}$$

$$ =0.5×0.2+0.5×0.1$$

$$ =0.15$$

Since $ρ\_{A,B}=0.05$, portfolio variance is:

$$σ\_{P}^{2}=x\_{A}^{2} .σ\_{A}^{2}+x\_{B}^{2} .σ\_{B}^{2}+2.x\_{A}.x\_{B}.σ\_{A,B}$$

and

$$σ\_{A,B}=ρ\_{A,B}.σ\_{A}.σ\_{B}$$

So:

$$σ\_{P}^{2}=x\_{A}^{2} .σ\_{A}^{2}+x\_{B}^{2} .σ\_{B}^{2}+2.x\_{A}.x\_{B}.ρ\_{A,B}.σ\_{A}.σ\_{B}$$

$$ =0.5^{2}×0.25^{2} + 0.5^{2}×0.15^{2} + $$

$$ + 2×0.5×0.5×0.05×0.25×0.15$$

$$σ\_{P}^{2}=0.0221875$$

$σ\_{P}=0.1490$ Notice that the standard deviation of P is less than both A and B’s standard deviation. This shows how diversification can lower risk.