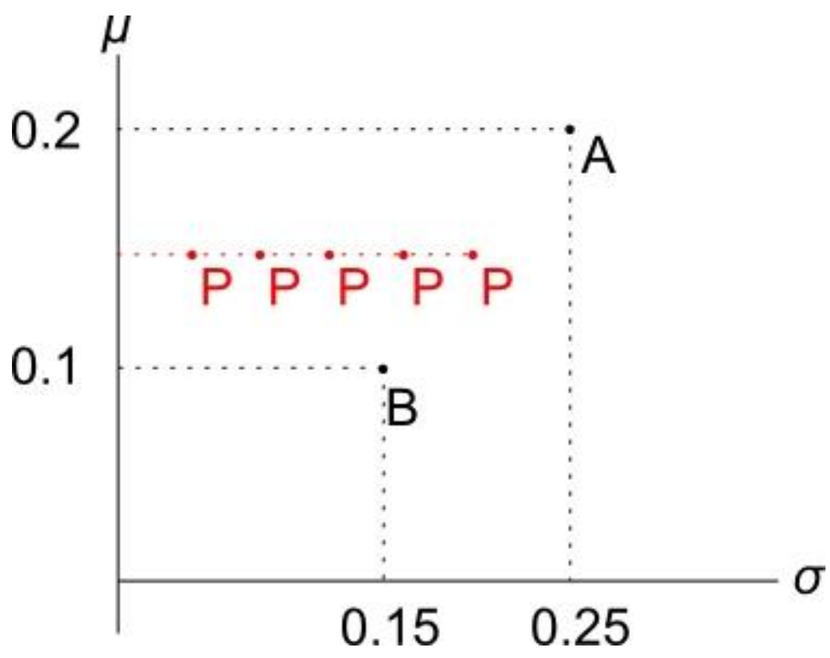
Portfolios and Diversification

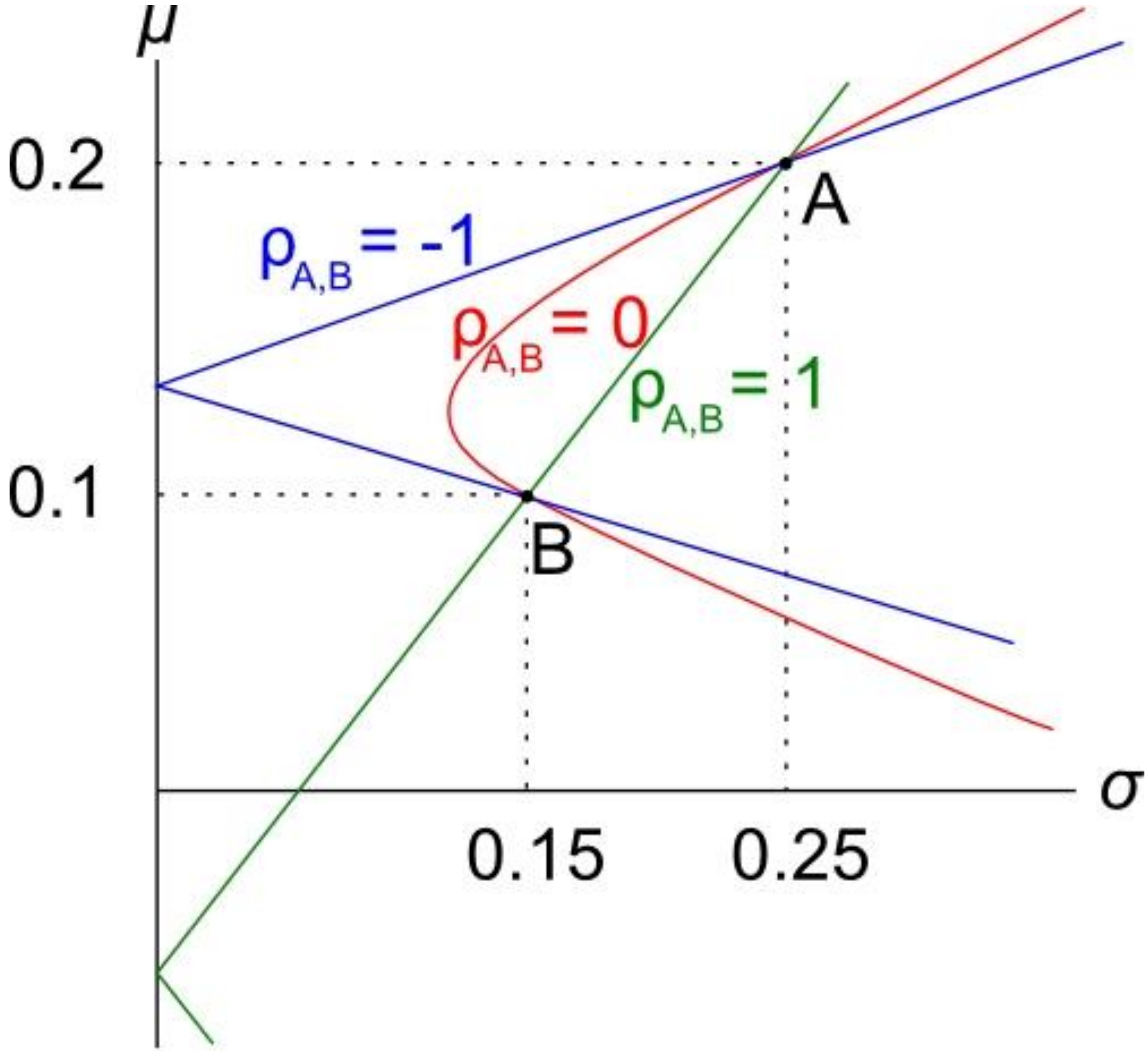
Let P be a \$100 portfolio with \$50 invested in A and \$50 in B.

- Therefore P is an equal-weighted portfolio in A and B. So:

  $$x_A = 0.5, \qquad x_B = 0.5$$

- Return will be exactly halfway at 0.15.
- Standard deviation is likely to be less than halfway (0.2) due to diversification - we didn't "put all of our eggs in one basket".
- So P will be somewhere along the dashed red line. The standard deviation of P depends on the **correlation** between A and B.

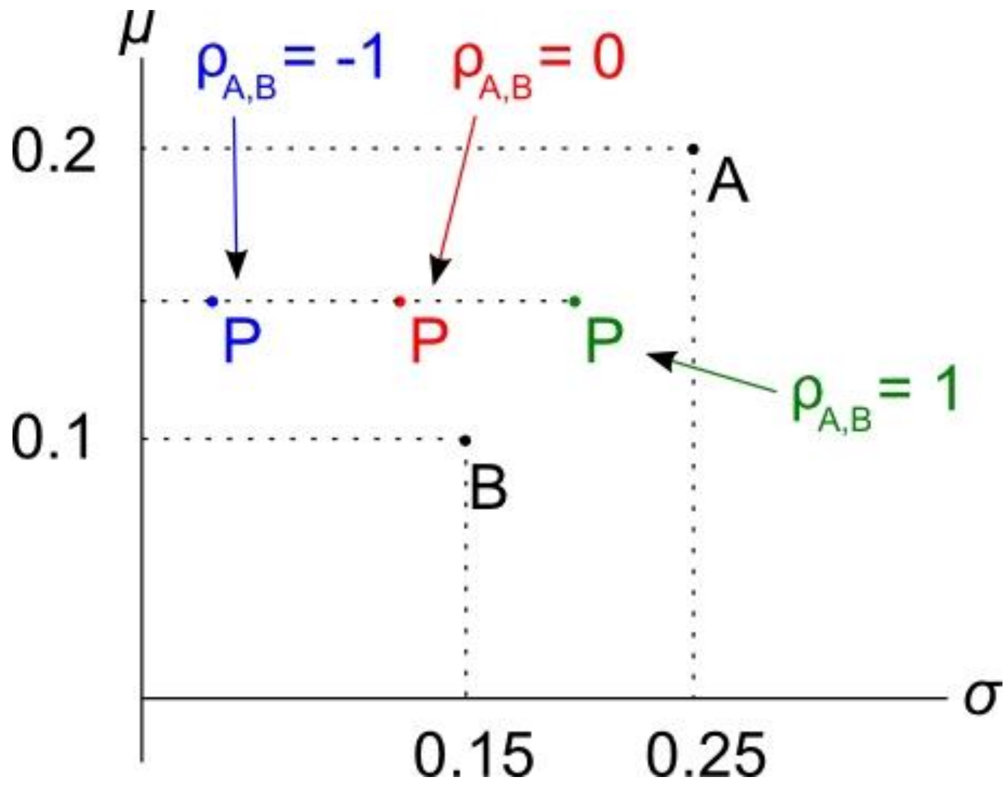




μ
0.2
0.1
A
B
ρA,B = -1
ρA,B = 0
ρA,B = 1
σ
0.15
0.25

How Correlation Affects Diversification

- $\rho_{A,B} = -1$, lots of diversification since A and B move in opposite directions.
- $\rho_{A,B} = 0$, diversification since A and B move independently and will sometimes cancel each other out.



- $\rho_{A,B} = 1$, no diversification at all since A and B move with each other in the same ratio. In this case $\sigma_P = 0.2$.

Portfolio Return and Variance

Portfolio return for n stocks with weights x:

$$r_P = x_1 . r_1 + x_2 . r_2 + \cdots + x_n . r_n = \sum_{i=1}^{n} (x_i . r_i)$$

Note that the weights must sum to one:

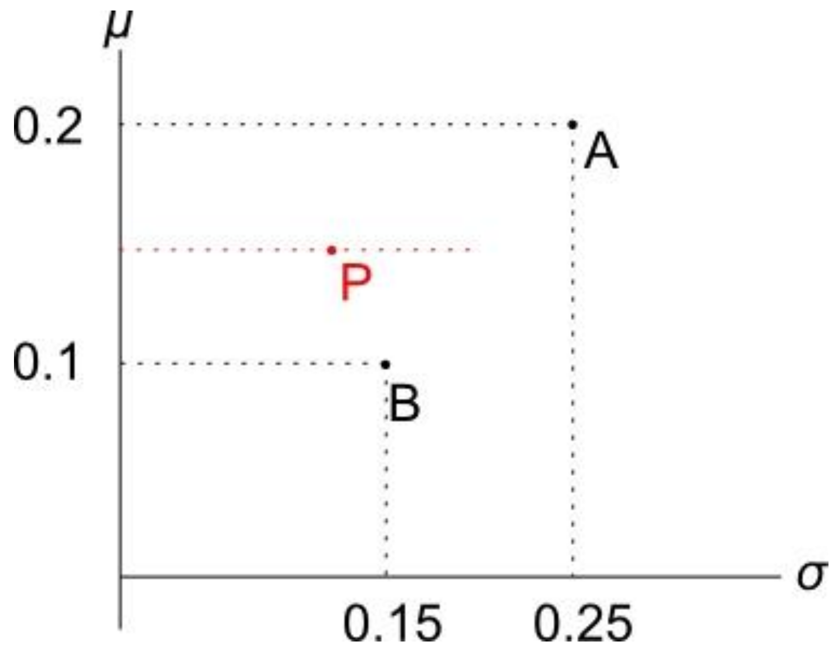
$$x_1 + x_2 + \cdots + x_n = 1$$

Portfolio variance for **2** stocks with weights x_1 and x_2:

$$\sigma_P{}^2 = x_1^2 . \sigma_1^2 + x_2^2 . \sigma_2^2 + 2. x_1 . x_2 . \sigma_{1,2}$$

Portfolio Return and Variance Example

Question: Portfolio P in the diagram has a weight of 0.5 in A and 0.5 in B. The correlation between A and B is 0.05. Find the return and variance of portfolio P.



Answer:

$$r_P = x_A.r_A + x_B.r_B$$

$$= 0.5 \times 0.2 + 0.5 \times 0.1$$

$$= 0.15$$

Since $\rho_{A,B} = 0.05$, portfolio variance is:

$$\sigma_P{}^2 = x_A^2 \,.\, \sigma_A^2 + x_B^2 \,.\, \sigma_B^2 + 2.\, x_A.\, x_B.\, \sigma_{A,B}$$

and

$$\sigma_{A,B} = \rho_{A,B}.\, \sigma_A.\, \sigma_B$$

So:

$$\sigma_P{}^2 = x_A^2 \,.\, \sigma_A^2 + x_B^2 \,.\, \sigma_B^2 + 2.\, x_A.\, x_B.\, \rho_{A,B}.\, \sigma_A.\, \sigma_B$$

$$= 0.5^2 \times 0.25^2 \;+\; 0.5^2 \times 0.15^2 \;+$$

$$+ 2 \times 0.5 \times 0.5 \times 0.05 \times 0.25 \times 0.15$$

$$\sigma_P{}^2 = 0.0221875$$

$\sigma_P = 0.1490$ Notice that the standard deviation of P is less than both A and B’s standard deviation. This shows how diversification can lower risk.