***Portfolios of 3 or More Stocks***

* Portfolios of only 2 stocks are restricted to a combination line. The combination line comprising stocks A and B is shown in black.
* After adding stock C, a whole area of portfolios are possible. There is a portfolio possibility ‘cloud’, which is the grey area in the graph.

***Constructing the 3+ Stock Markowitz Bullet***

This requires a formula for multi-stock portfolio variance.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | $$x\_{1}$$ | $$x\_{2}$$ | $$x\_{3}$$ | $$x\_{4}$$ |  | The grey-shaded part of the table is called the variance-covariance matrix. It has the variance of each stock along the diagonal, and covariances elsewhere.Note that $σ\_{1,2}=σ\_{2,1}$ and $σ\_{1,1}=σ\_{1}^{2}$ |
| $$x\_{1}$$ | $$σ\_{1}^{2}$$ | $$σ\_{1,2}$$ | $$σ\_{1,3}$$ | $$σ\_{1,4}$$ |
| $$x\_{2}$$ | $$σ\_{2,1}$$ | $$σ\_{2}^{2}$$ | $$σ\_{2,3}$$ | $$σ\_{2,4}$$ |
| $$x\_{3}$$ | $$σ\_{3,1}$$ | $$σ\_{3,2}$$ | $$σ\_{3}^{2}$$ | $$σ\_{3,4}$$ |
| $$x\_{4}$$ | $$σ\_{4,1}$$ | $$σ\_{4,2}$$ | $$σ\_{4,3}$$ | $$σ\_{4}^{2}$$ |

Portfolio variance is equal to the sum of each term in the variance-covariance matrix multiplied by its corresponding two weights.

$$σ\_{P}^{2}=x\_{1}x\_{1}σ\_{1,1}+x\_{1}x\_{2}σ\_{1,2}+…+x\_{4}x\_{3}σ\_{4,3}+x\_{4}x\_{4}σ\_{4,4}$$

After collecting like terms and re-arranging, we have the 4-stock portfolio variance equation:

$$σ\_{P}^{2}=x\_{1}^{2}σ\_{1}^{2} +x\_{2}^{2}σ\_{2}^{2} +x\_{3}^{2}σ\_{3}^{2} +x\_{4}^{2}σ\_{4}^{2}+$$

$$ 2x\_{1}x\_{2}σ\_{1,2}+2x\_{1}x\_{3}σ\_{1,3}+2x\_{1}x\_{4}σ\_{1,4}+$$

$$ 2x\_{2}x\_{3}σ\_{2,3}+2x\_{2}x\_{4}σ\_{2,4}+$$

$$ 2x\_{3}x\_{4}σ\_{3,4}$$

Here’s the 3-stock portfolio variance equation:

$$σ\_{P}^{2}=x\_{1}^{2}σ\_{1}^{2} +x\_{2}^{2}σ\_{2}^{2} +x\_{3}^{2}σ\_{3}^{2}$$

$$ 2x\_{1}x\_{2}σ\_{1,2}+2x\_{1}x\_{3}σ\_{1,3}+$$

$$ 2x\_{2}x\_{3}σ\_{2,3}$$