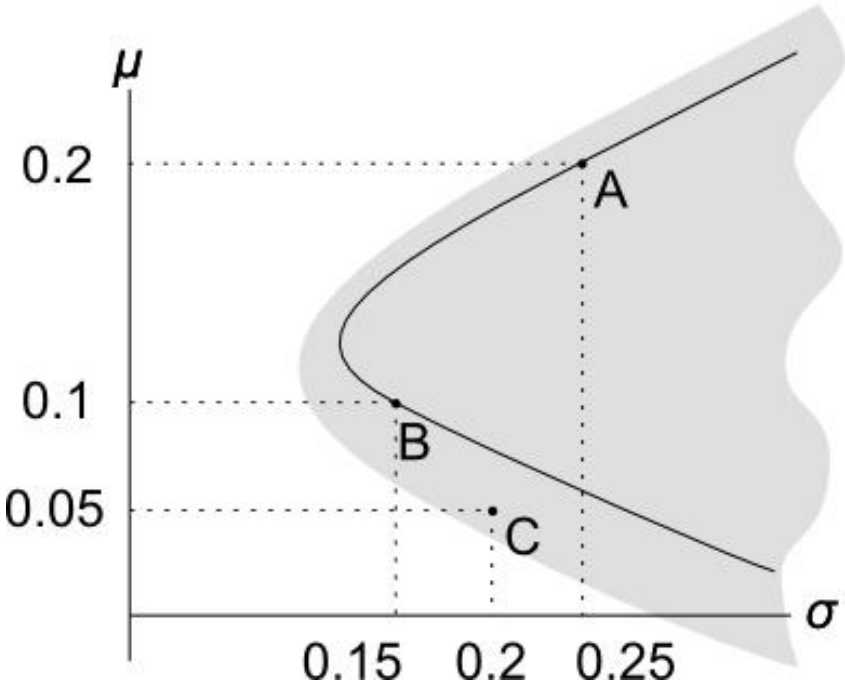
Portfolios of 3 or More Stocks

- Portfolios of only 2 stocks are restricted to a combination line. The combination line comprising stocks A and B is shown in black.

- After adding stock C, a whole area of portfolios are possible. There is a portfolio possibility 'cloud', which is the grey area in the graph.



Constructing the 3+ Stock Markowitz Bullet

This requires a formula for multi-stock portfolio variance.

	x_1	x_2	x_3	x_4
x_1	${\sigma_1}^2$	$\sigma_{1,2}$	$\sigma_{1,3}$	$\sigma_{1,4}$
x_2	$\sigma_{2,1}$	${\sigma_2}^2$	$\sigma_{2,3}$	$\sigma_{2,4}$
x_3	$\sigma_{3,1}$	$\sigma_{3,2}$	${\sigma_3}^2$	$\sigma_{3,4}$
x_4	$\sigma_{4,1}$	$\sigma_{4,2}$	$\sigma_{4,3}$	${\sigma_4}^2$

The grey-shaded part of the table is called the variance-covariance matrix. It has the variance of each stock along the diagonal, and covariances elsewhere.

Note that $\sigma_{1,2} = \sigma_{2,1}$ and $\sigma_{1,1} = \sigma_1^2$

Portfolio variance is equal to the sum of each term in the variance-covariance matrix multiplied by its corresponding two weights.

$${\sigma_P}^2 = x_1 x_1 \sigma_{1,1} + x_1 x_2 \sigma_{1,2} + \cdots + x_4 x_3 \sigma_{4,3} + x_4 x_4 \sigma_{4,4}$$

After collecting like terms and re-arranging, we have the 4-stock portfolio variance equation:

$$\sigma_P{}^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + x_3^2\sigma_3^2 + x_4^2\sigma_4^2 + 2x_1x_2\sigma_{1,2} + 2x_1x_3\sigma_{1,3} + 2x_1x_4\sigma_{1,4} + 2x_2x_3\sigma_{2,3} + 2x_2x_4\sigma_{2,4} + 2x_3x_4\sigma_{3,4}$$

Here's the 3-stock portfolio variance equation:

$$\sigma_P{}^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + x_3^2\sigma_3^2 + 2x_1x_2\sigma_{1,2} + 2x_1x_3\sigma_{1,3} + 2x_2x_3\sigma_{2,3}$$