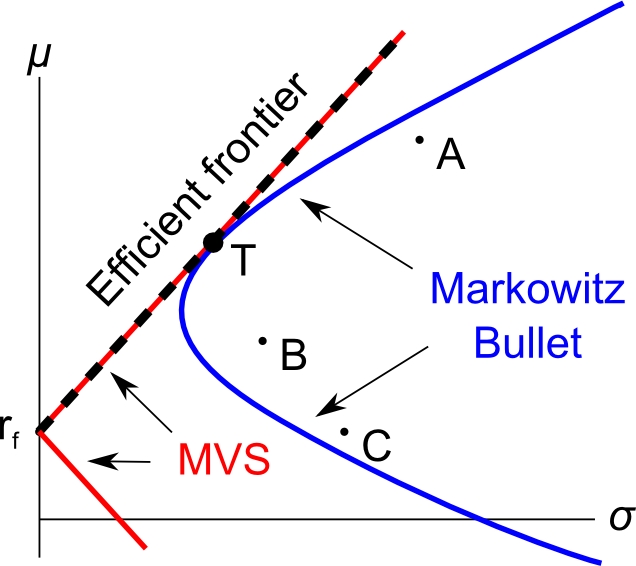
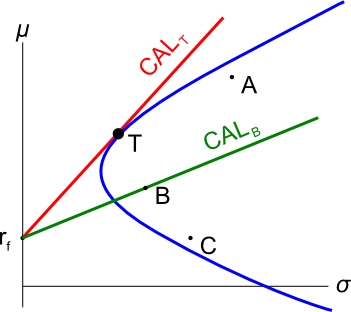
***The Risk Free Rate (rf) and the Minimum Variance Set of All Assets***

* Risk-free securities have zero standard deviation of returns. Government bonds (also called Treasuries) are assumed to be risk free securities.
* The return of the risk-free security is referred to as . It’s also used to refer to the security itself.
* When is included, the **new MVS becomes a line** from through the tangency portfolio (T) on the Markowitz bullet.
* is a constant, so it has zero variance, and zero covariance with other securities.
* , since is a constant.

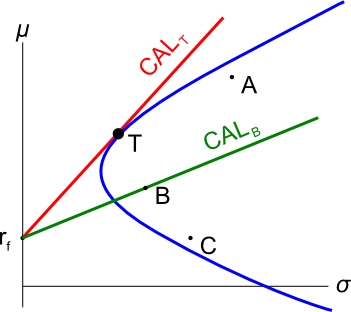
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***The Tangency Portfolio***

* The tangency portfolio T is the only risky portfolio worth investing in. It is comprised of the stocks A, B and C.
* For any level of risk (standard deviation), the highest return possible can be achieved by investing in T and .
* Lines from through any portfolio are called Capital Allocation Lines.
* The CAL through T has the best risk-return trade off. It has the steepest gradient (rise/run).
* The gradient of the CAL is also called the Sharpe ratio.

***Sharpe Ratio***

The **Sharpe ratio** (S) of a stock is the gradient of the line from through the stock. It is the gradient of the stock’s CAL.

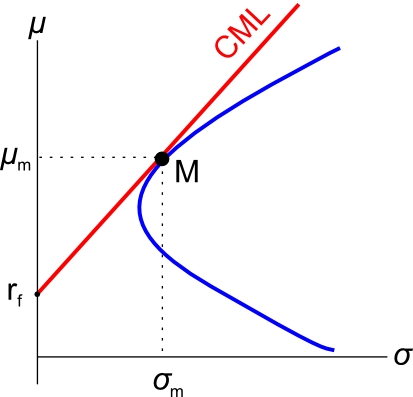


Where is the Sharpe ratio of stock ‘i’, is its expected return and is its standard deviation.

In the diagram, portfolio T’s Sharpe ratio is greater than stock B’s since the CAL through T is steeper. Therefore portfolio T is preferable to stock B.

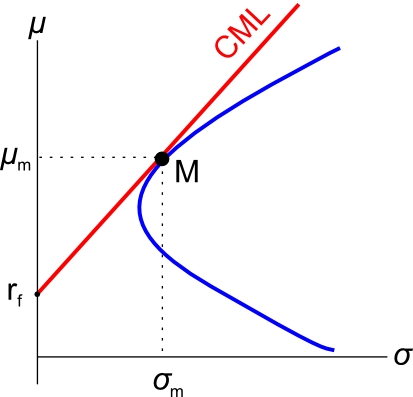
***The Market Portfolio***

* The market portfolio M is the tangency portfolio of **all** risky assets.



* The line through M and is called the Capital Market Line (CML).
* The CML has the steepest gradient, therefore the market portfolio has the highest Sharpe ratio.

***Equation of the Capital Market Line (CML)***

**

This is easy to see since the:

* y-axis is expected return
* x-axis is standard deviation
* y-intercept is
* gradient between and M is

So in form:

***Calculation Example***

**Question**: Assume a 3-stock world consisting of A, B and C, as well as the risk free security. The market portfolio has been calculated to have weights 1/3 in each of A, B and C. The risk free rate , the market return is and the market’s standard deviation is .

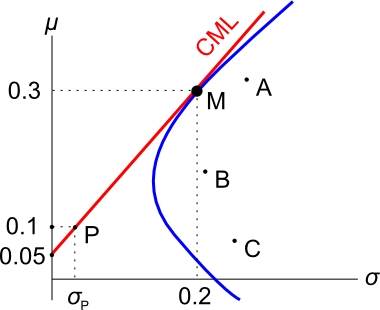
Find the weights in stocks A, B, C and which makes an efficient portfolio (P) with a return of . Also find this portfolio’s standard deviation .

**Answer**: An efficient portfolio has minimum variance (or st. dev.) for a given return. All portfolios on the CML are efficient. Therefore we need only consider investing in the market portfolio (M) and the risk free rate ().

To find the weights we need to invest in M and , we will use the portfolio return equation, with a target portfolio return of 0.1:

Now we’re stuck since we have 2 unknowns ( and ) and only one equation so we can’t find either of the weights. But there is another equation, the ‘sum of the weights equals one’:

Substitute this into the portfolio return equation to get:



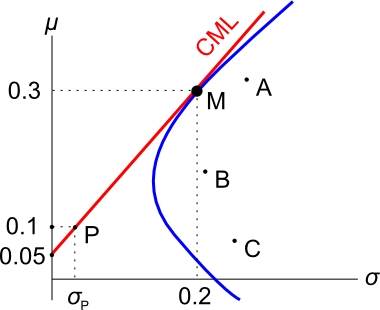
So,

This makes sense since the target return of 0.1 is closer to so it should have a larger weight in than M.

Since M is 1/3 in each of A, B and C, the weights in A, B and C are simply 1/3 of the weight in M:

To find this efficient portfolio’s standard deviation, we could use the 2-stock portfolio variance equation with and M and the weights we just found, together with the fact that the covariance of with is zero since is a constant:

But another faster method is to use the CML equation instead:



Where is the return of our efficient portfolio of 0.1, and is the variable we are trying to find.