***Capital Asset Pricing Model (CAPM)***

The CAPM separates total variance into two types:

**Systematic variance**

* Also called market or undiversifiable variance. This risk can not be avoided and affects all assets except treasury bonds.
* Caused by macro-economic events such as interest rate changes, the government’s budget, a financial boom or crisis, a natural disaster, a currency crisis, or a statistics release.
* Only systematic risk should affect an asset’s expected return or price since it cannot be diversified away.

**Idiosyncratic variance**

* Also called residual, firm-specific, diversifiable, non-systematic or non-market variance.
* Caused by events such as an oil company’s discovery of a new oil field, the death of a firm’s CEO, tax breaks for a specific industry, or fraud or rogue-trading losses at a bank.
* Idiosyncratic risk can be diversified away to zero by investing in a large enough portfolio of assets. Therefore it should not affect an asset’s expected return or price.

***CAPM – Beta (β)***

Systematic risk can be measured using beta ($β$).

$$β\_{i}= \frac{σ\_{i,M}}{σ\_{M}^{2}}=\frac{cov(r\_{i}, r\_{M})}{var(r\_{M})}=ρ\_{i,M}\frac{σ\_{i}}{σ\_{M}}=correl\left(r\_{i}, r\_{M}\right).\frac{stdev\left(r\_{i}\right)}{stdev\left(r\_{m}\right)}$$

Where $β\_{i}$ is the beta of stock i, $r\_{i}$ is the return of stock i and $r\_{M}$ is the return of the market portfolio. The higher the beta of a stock, the more sensitive it is to movements in the market.

**Interpretation:** If a stock’s beta is **2**, then a sudden **1**% increase in the price of the market portfolio would be expected to cause a **2**% increase in the stock’s price.

$$β\_{i}= \frac{σ\_{i,M}}{σ\_{M}^{2}}=\frac{cov(r\_{i}, r\_{M})}{var(r\_{M})}$$

The beta of the market portfolio $β\_{M}$ equals one.

* This makes sense since the covariance of $r\_{M}$ with itself equals its variance, $\frac{cov\left(r\_{M}, r\_{M}\right)}{var\left(r\_{M}\right)}=\frac{var\left(r\_{M}\right)}{var\left(r\_{M}\right)}=1$.

The beta of the risk free security is zero.

* This makes sense since the risk free rate is a constant and the covariance of a constant with any variable is zero.

Note: variance ($σ^{2}$) can also be used to measure systematic risk as well as beta ($β$). The relationship, which we’ll examine later, is: $σ\_{i syst}^{2}=β\_{i}^{2}.σ\_{M}^{2}$

***The CAPM Equation***

$$r\_{i}=r\_{f}+β\_{i}\left(r\_{M}-r\_{f}\right)+ε\_{i}$$

Where: $r\_{i}$ is the return of stock i, it’s a variable,

$r\_{f}$ is the risk-free rate, it’s a constant,

$β\_{i}= \frac{σ\_{i,M}}{σ\_{M}^{2}}$, which is the systematic risk factor of stock i, it’s a constant,

$r\_{M}$ is the market portfolio’s return, it is a variable and is the **source of market risk**.

$ε\_{i}$ is the residual return of stock i. It is the unpredictable random error which averages zero. It is the **source of idiosyncratic risk**. It’s a variable.

***The Security Market Line (SML) Equation***

Taking the expectations of both sides of the CAPM equation, which is the same as taking the average,

$$μ\_{i}=r\_{f}+β\_{i}\left(μ\_{M}-r\_{f}\right)$$

Where: $μ\_{i}$ is the expected or average return of stock i. It can also be written as $E(r\_{i})$. Note that it’s ok to just use $r$ instead of $μ$ or $E(r)$ in this course.

$μ\_{M}$ is the expected return of the market. It can also be written as $E(r\_{m})$.

$r\_{f}$ is a constant so $E\left(r\_{f}\right)=μ\_{rf}=r\_{f}$, so we just write $r\_{f}$.

Notice that the error term $ε\_{i}$, also known as the residual, drops out because its average is zero. ie, $E\left(ε\_{i}\right)=0$.

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| **SML Equation & Graph**$$μ=r\_{f}+β\left(μ\_{M}-r\_{f}\right)$$ | **CML Equation & Graph**$$μ=r\_{f}+σ\left(\frac{μ\_{m}-r\_{f}}{σ\_{m}}\right)$$ |
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