Capital Asset Pricing Model (CAPM)

The CAPM separates total variance into two types:

Systematic variance

- Also called market or undiversifiable variance. This risk can not be avoided and affects all assets except treasury bonds.
- Caused by macro-economic events such as interest rate changes, the government's budget, a financial boom or crisis, a natural disaster, a currency crisis, or a statistics release.
- Only systematic risk should affect an asset's expected return or price since it cannot be diversified away.

Idiosyncratic variance

- Also called residual, firm-specific, diversifiable, non-systematic or non-market variance.
- Caused by events such as an oil company's discovery of a new oil field, the death of a firm's CEO, tax breaks for a specific industry, or fraud or rogue-trading losses at a bank.
- Idiosyncratic risk can be diversified away to zero by investing in a large enough portfolio of assets. Therefore it should not affect an asset's expected return or price.

CAPM – Beta (β)

Systematic risk can be measured using beta (β).

$$\beta_i = \frac{\sigma_{i,M}}{\sigma_M{}^2} = \frac{cov(r_i, r_M)}{var(r_M)} = \rho_{i,M}\frac{\sigma_i}{\sigma_M} = correl(r_i, r_M).\frac{stdev(r_i)}{stdev(r_m)}$$

Where β_i is the beta of stock i, r_i is the return of stock i and r_M is the return of the market portfolio. The higher the beta of a stock, the more sensitive it is to movements in the market.

Interpretation: If a stock's beta is **2**, then a sudden **1**% increase in the price of the market portfolio would be expected to cause a **2**% increase in the stock's price.

$$\beta_i = \frac{\sigma_{i,M}}{{\sigma_M}^2} = \frac{cov(r_i, r_M)}{var(r_M)}$$

The beta of the market portfolio β_M equals one.

- This makes sense since the covariance of r_M with itself equals its variance, $\frac{cov(r_M, r_M)}{var(r_M)} = \frac{var(r_M)}{var(r_M)} = 1$.

The beta of the risk free security is zero.

- This makes sense since the risk free rate is a constant and the covariance of a constant with any variable is zero.

Note: variance (σ^2) can also be used to measure systematic risk as well as beta (β). The relationship, which we'll examine later, is: ${\sigma_{i\,syst}}^2 = {\beta_i}^2 . {\sigma_M}^2$

The CAPM Equation

$$r_i = r_f + \beta_i(r_M - r_f) + \varepsilon_i$$

Where: r_i is the return of stock i, it's a variable,

r_f is the risk-free rate, it's a constant,

$\beta_i = \frac{\sigma_{i,M}}{{\sigma_M}^2}$, which is the systematic risk factor of stock i, it's a constant,

r_M is the market portfolio's return, it is a variable and is the **source of market risk**.

ε_i is the residual return of stock i. It is the unpredictable random error which averages zero. It is the **source of idiosyncratic risk**. It's a variable.

The Security Market Line (SML) Equation

Taking the expectations of both sides of the CAPM equation, which is the same as taking the average,

$\mu_i = r_f + \beta_i(\mu_M - r_f)$

Where: μ_i is the expected or average return of stock i. It can also be written as $E(r_i)$. Note that it's ok to just use r instead of μ or $E(r)$ in this course.

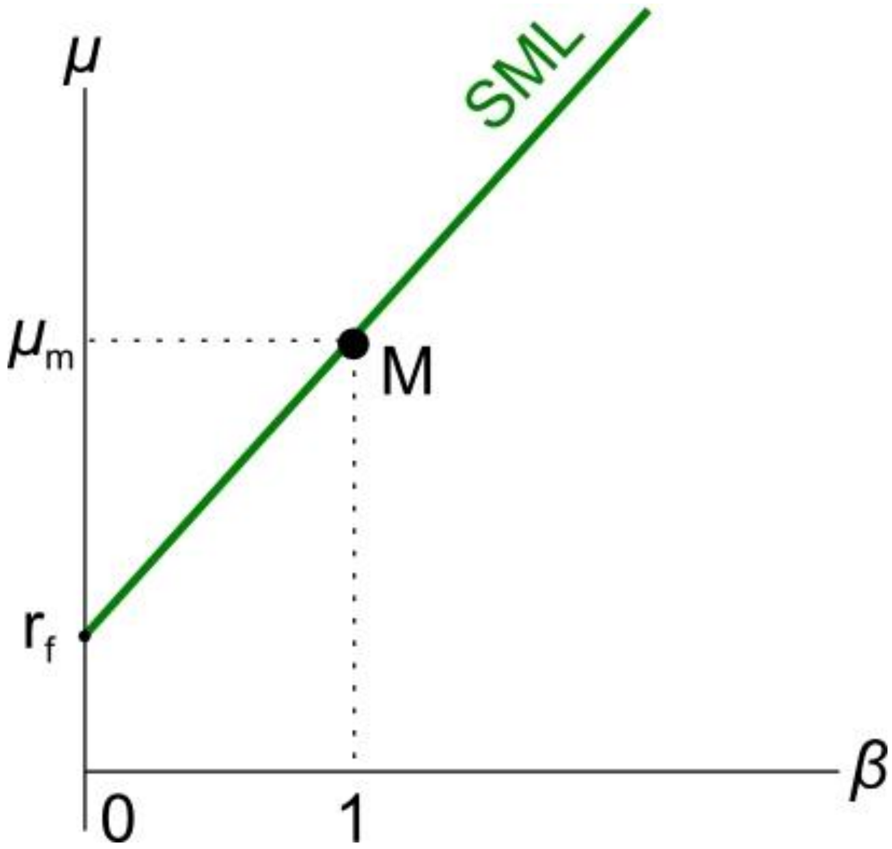
μ_M is the expected return of the market. It can also be written as $E(r_m)$.

r_f is a constant so $E(r_f) = \mu_{rf} = r_f$, so we just write r_f.

Notice that the error term ε_i, also known as the residual, drops out because its average is zero. ie, $E(\varepsilon_i) = 0$.

SML Equation & Graph

$$\mu = r_f + \beta(\mu_M - r_f)$$



CML Equation & Graph

$$\mu = r_f + \sigma\left(\frac{\mu_m - r_f}{\sigma_m}\right)$$