Modern Portfolio Theory – A Brief History

1950's - The foundations of modern portfolio theory were built by Harry Markowitz with his work on diversification and mean-variance efficiency.

1960's – Sharpe, Lintner and Mossin extended Markowitz's work in the form of the Capital Asset Pricing Model (CAPM). Interesting interview with Bill Sharpe: http://web.stanford.edu/~wfsharpe/art/djam/djam.htm

1976 - Stephen Ross derived similar results to the CAPM in the more general Arbitrage Pricing Theory (APT). The APT requires only 3 assumptions compared to the many assumptions required in the CAPM.

Putting the CAPM in Perspective

So far we have actually been looking at the original Sharpe-Lintner-Mossin CAPM which is a single factor model that's static since the betas are assumed to remain constant over time.

The CAPM's only factor is the market risk premium $\left(r_M - r_f\right)$, where r_M is a variable and r_f is a constant.

Now that we've discussed some of the ideas and mathematics behind the CAPM, let's look back at how the CAPM extends Markowitz's Mean-Variance Framework.

Drawbacks of the Markowitz Mean-Variance Approach

1. Cannot be used to calculate the **price** or **expected return** of individual stocks. Only explains how to make an efficient *portfolio* with the minimum variance for a given return. Returns of the individual stocks are needed as inputs into the model.
2. For each new stock that is added to the model, the covariance of that stock with every other stock needs to be calculated.
3. Estimates of covariances are likely to change over time so sample estimates are likely to be unreliable.
4. No concept of systematic (undiversifiable or market) risk.

Factor Pricing Models

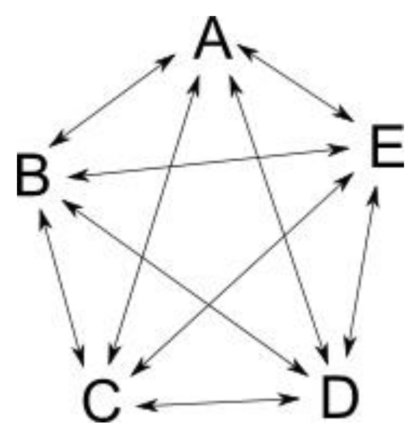
Factor pricing models simplify problem 2 by assuming that stocks are affected by common risk factors, such as returns on the market portfolio, GDP growth, and the unemployment rate for example.

When a stock is added to a factor model, only that stock's covariance with each factor needs to be calculated. For the single factor CAPM, the covariance of the stock with the market portfolio M gives the numerator of the beta, and the denominator is just the market's variance $\left(\beta_i = \frac{\sigma_{i,M}}{{\sigma_M}^2}\right)$.

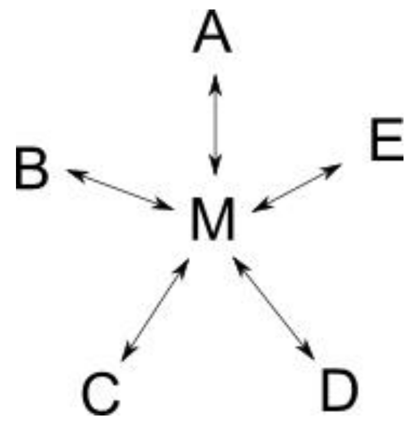
The CAPM and APT are both factor models and there are single-factor and multi-factor versions of each.

Covariances Between Stocks

In Markowitz's mean-variance framework, the covariance of each stock (A, B, C, D, E) with all others must be calculated.



But in a factor model such as the CAPM, only the covariance of each stock with M is needed. This implies that the only relationship between say A and B is through M. So if the CAPM is true, then residual returns should be independent of each other:



$$cov(\varepsilon_A, \varepsilon_B) = 0.$$

This not required in Markowitz's mean-variance approach.

The CAPM

- The single-factor version of the CAPM:
  $r_i = r_f + \beta_i(r_M - r_f) + \varepsilon_i$
- The only factor is the market risk premium $(r_M - r_f)$, where r_M is a variable and r_f is a constant.
- Assumes that stock returns are only determined by changes in the market return r_M and random firm-specific changes in the residual return ε_i.
- Therefore the only relationship between the returns of two stocks A and B is through changes in the market rate of return r_M, so the stocks' residual returns should be independent: $cov(\varepsilon_A, \varepsilon_B) = 0$

CAPM Formulas

$r_i = r_f + \beta_i(r_M - r_f) + \varepsilon_i$ and $\mu_i = r_f + \beta_i(\mu_M - r_f)$

$cov(\varepsilon_A, \varepsilon_B) = 0$

$$\beta_i = \frac{\sigma_{i,M}}{{\sigma_M}^2}$$

From the above, the following equations can be derived:

$${\sigma_{i,total}}^2 = {\beta_i}^2 . {\sigma_M}^2 + {\sigma_{i,\varepsilon}}^2$$

$$\sigma_{1,2} = \beta_1 . \beta_2 . {\sigma_M}^2$$

$$\beta_P = x_1\beta_1 + x_2\beta_2 + \cdots + x_n\beta_n$$