

# *Expected returns and probabilities*

Probabilities represent the chance of something happening. Some useful rules:

- The sum of the probabilities of all possible outcomes is always 1 (=100%).
- 'Or' means sum the probabilities.
- 'And' means multiply the probabilities.

**Question:** At university you can pass or fail a subject. If the probability of failing is 10%, what is the chance of passing the subject?

**Answer:** You can pass *or* fail, and the 'or' means add the probabilities. The sum of the probabilities of the complete

set of outcomes is always one, so the probability of failing or passing must be one. Let the probability of passing be  $p_{pass}$  and the probability of failing be  $p_{fail}$ .

$$p_{pass} + p_{fail} = 1$$

$$p_{pass} + 0.1 = 1$$

$$p_{pass} = 1 - 0.1 = 0.9$$

So there's a 90% chance of passing a single subject.

**Question:** In a 3 year university degree with 4 subjects per semester and 2 semesters per year, a student will complete 24 ( $=3*2*4$ ) subjects. If the chance of failing a single subject is 10%, what is the chance of passing every single subject? Assume that you have average ability and motivation.

**Answer:** Passing every subject means that you must pass the first one and the second and the third and so on. Since it's 'and', the probabilities must be multiplied. The chance of passing a single subject is 90%. So the chance of passing all 24 subjects is:

$$\begin{aligned} p_{\text{pass all 24 subjects}} &= p_{\text{pass},1} \times p_{\text{pass},2} \times \dots \times p_{\text{pass},24} \\ &= 0.9 \times 0.9 \times \dots \times 0.9 = 0.9^{24} \\ &= 0.079766443 \approx 8\% \end{aligned}$$

## *Expected returns, uncertainty and probabilities*

The expected return of an asset when there are different possible states of the world (good, ok, bad, and so on) is the sum of the return in each state of the world multiplied by the probability.

## *Calculation example*

**Question:** Find the expected return on the stock market given the below information about stocks' returns in different possible states of the economy.

Stock Returns in Different States of the Economy		
State of economy	Probability	Return
Boom	0.3	0.6
Normal	0.5	0.1
Bust	0.2	-0.5

**Answer:** the expected return 'E(r)' or  $\mu$  is equal to:

$$\begin{aligned} E(r) &= p_1 \cdot r_1 + p_2 \cdot r_2 + \cdots + p_n \cdot r_n \\ &= 0.3 \times 0.6 + 0.5 \times 0.1 + 0.2 \times -0.5 \\ &= 0.13 = 13\% \end{aligned}$$