Continuously compounded returns

Continuously compounded (cc) returns are defined as:

$$r_{cc\ 0\rightarrow 1} = \ln\left(\frac{p_1}{p_0}\right) = \log_e\left(\frac{p_1}{p_0}\right)$$

Annual continuously compounded returns are equivalent to annualised percentage rates (APR's) with a compounding period that is not per month, nor per day nor per second but infinitely small or continuous.

Converting Between Continuous and Effective Returns

Continuously Compounding (r_{cc}) to Effective Discretely Compounding (r_{eff}) Return:

$$r_{eff} = e^{r_{cc}} - 1 = \exp(r_{cc}) - 1$$

Effective Discretely Compounding $\left(r_{eff}\right)$ to Continuously Compounding (r_{cc}) Return:

$$r_{cc} = \ln\left(1 + r_{eff}\right) = \log_e\left(1 + r_{eff}\right)$$

Note that the above equations assume that each return is measured over the same time. So both the r_{cc} and r_{eff} are annual, for example.

Different Return Quotations Equivalent to an Effective Annual Rate of 10%

Quote type	Return (%pa)	Symbol
Effective annual rate	10	$r_{eff\ annual}$
APR compounding per annum	10	$r_{apr\ comp\ annually}$
APR compounding semi-annually	9.761769634	$r_{apr\ comp\ 6mth}$
APR compounding quarterly	9.645475634	$r_{apr\ comp\ quarterly}$
APR compounding monthly	9.568968515	$r_{apr\ comp\ monthly}$
APR compounding daily	9.532279763	$r_{apr\ comp\ daily}$
APR compounding hourly	9.531070550	$r_{apr\ comp\ hourly}$
APR compounding per minute	9.531018861	$r_{apr\ comp\ per\ minute}$
APR compounding per second	9.531018227	$r_{apr\ comp\ per\ second}$
Continuously compounded annual rate	9.531017980	$r_{cc\ annual}$

Adjusting for Time: Compounding Up and Down

Turning a continuously compounded **monthly** return into an **annual** return is very easy, just multiply by 12. This is compounding up:

$$r_{cc\ annual} = 12 \times r_{cc\ monthly}$$

To compound down from an annual rate to a daily rate where there are 365 days in the year, just divide by 365.

$$r_{cc\ daily} = \frac{r_{cc\ annual}}{365}$$

Present Values with Continuously Compounding Returns

Since $\left(1 + r_{eff}\right)^{t}$ is equivalent to $e^{r_{cc}.t}$, then:

$$PV(single\ cash\ flow) = V_0 = \frac{C_t}{e^{r_{cc}.t}}$$

$$= C_t.\, e^{-r_{cc}.t}$$

$$= \frac{C_t}{\exp(r_{cc}.\, t)}$$

$$= \mathrm{C_t}.\, \exp(-r_{cc}.\, t)$$

Calculation Example: Present Value of a Single Cash Flow

Question: What is the present value of $100 received in 5 years when continuously compounded interest rates are 8% pa?

Answer:

$$V_0 = \frac{C_t}{e^{r_{cc} \cdot t}}$$

$$= \frac{100}{e^{0.08 \times 5}}$$

$$= 67.0320046$$

Calculation Example: <u>Future</u> Value of a Single Cash Flow

Question: You have $100 in the bank. Interest rates are 8% pa. How much will you have in the bank after 5 years?

Answer:

$$V_t = C_0 . e^{r_{cc}.t}$$

$$= 100 \times e^{0.08 \times 5}$$

$$= 149.1824698$$

Total, Capital and Income returns with continuous compounding

$$r_{cc\,total} = r_{cc\,capital} + r_{cc\,income}$$

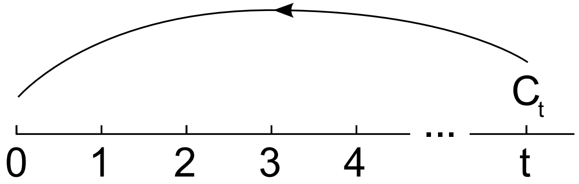
This also holds after raising both sides to the power of Euler's number 'e':

$$e^{r_{cc\,total}} = e^{r_{cc\,capital} + r_{cc\,income}}$$

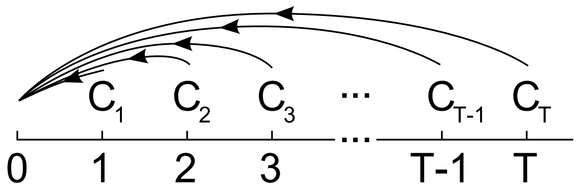
$$= e^{r_{cc\,capital}} . e^{r_{cc\,income}}$$

Present Value Formulas: Cont. Comp. Rates

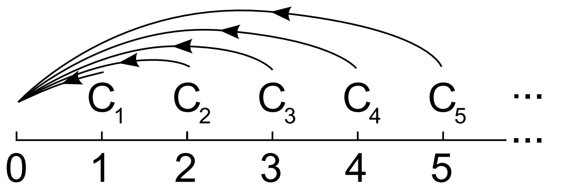
$$PV(single\ cash\ flow) = V_0 = \frac{C_t}{e^{r_{cc}.t}}$$



$$PV(annuity) = V_0 = \frac{C_1}{e^{r_{cc}} - 1}\left(1 - \frac{1}{e^{r_{cc}.t}}\right)$$

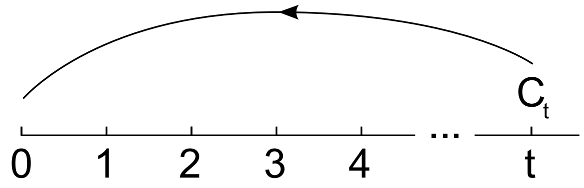


$$PV(perpetuity) = V_0 = \frac{C_1}{e^{r_{cc}} - e^{g_{cc}}}$$

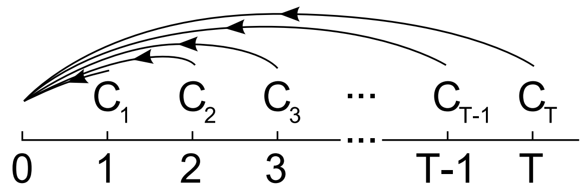


Present Value Formulas: Effective Rates

$$PV(single\ cash\ flow) = V_0 = \frac{C_t}{(1 + r_{eff})^t}$$



$$PV(annuity) = V_0 = \frac{C_1}{r_{eff}}\left(1 - \frac{1}{(1+r_{eff})^T}\right)$$



$$PV(perpetuity) = V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$