Optimal or Minimum-Variance Hedging

Basis risk occurs when trying to hedge the risk of the underlying asset using:

- cross-hedging; or
- hedging using futures that expire on non-ideal dates.

Basis risk corresponds to variance in the hedged portfolio.

To reduce basis risk, the 'optimal hedge ratio' can be calculated, which minimises the hedged portfolio's variance.

The hedge ratio can be used to calculate the number of futures contracts to buy or sell.

Optimal Hedge Ratio

The optimal hedge ratio (h) is:

$$h = \rho_{S_T,F_{0,T}} . \frac{\sigma_{S_T}}{\sigma_{F_{0,T}}} = correl(S_T, F_{0,T}) . \frac{sd(S_T)}{sd(F_{0,T})}$$

The hedge ratio can be interpretted as:

"A 1% increase in the futures or forward price is expected to cause a h% increase in the underlying asset price".

Number of Contracts

To calculate the optimal number of forward or futures contracts (n) to buy:

$$n_{no\ tailing\ forwards} = h.\frac{Q_S}{Q_F}$$

$$n_{tailing\ futures} = h.\frac{V_S}{V_F} = h.\frac{Q_S.S_t}{Q_F.F_{t,T}}$$

Futures contracts are settled daily so they must be ‘tailed’, which takes into account the daily gains and losses in the margin account.

Forward contracts are not tailed since there’s no margin account.

The (non-optimal) hedge ratio that reduces correlation to some accepted level $\rho^*_{S_T,F_{0,T}}$ is:

$$h_{non-optimal} = (\rho_{S_T,F_{0,T}} - \rho^*_{S_T,F_{0,T}}) \cdot \frac{\sigma_{S_T}}{\sigma_{F_{0,T}}}$$

Equity Index Futures: Optimal Hedge Ratio

When hedging using stock index forwards or futures, the hedge ratio is the CAPM beta, so:

$$n_{tailing\ futures} = B_S . \frac{V_S}{V_F}$$

If a desired beta of B_S^* is wanted to partly-hedge a well-diversified stock portfolio, then the number of equity index futures contracts to sell is:

$$n_{tailing\ futures,non-optimal} = (B_S - B_S^*) . \frac{V_S}{V_F}$$

Where B_S is the existing beta of the portfolio to be hedged and B_S^* is the target portfolio beta.