Debt

The debt markets are far more complicated and filled with jargon than the equity markets.

Fixed coupon bonds, loans and bills will be our focus in this course.

It's important to be aware of all of the different ways debt can be classified. For example:

- Retail or wholesale
- Floating or fixed coupons
- Secured and unsecured
- Premium, par or discount
- Call-able, put-able or vanilla

- Seniority: Super-senior, senior, mezzanine, subordinated, super-subordinated.
- **Securities** such as bonds and notes which are fungible and saleable (negotiable). **Instruments,** a more broad category, which includes securities and also loans and bills which are not fungible or negotiable.
- Rated and un-rated.

Some strange things about debt markets:

- Risk is not usually quoted as a standard deviation or variance or beta, but as a 'rating'.
- Rating agencies S&P and Fitch use ratings: AAA, AA+, AA, AA-, A+, A, A-, BBB+, BBB, BBB-, BB+ and so on.

- Moodys uses ratings: Aaa, Aa1, Aa2, Aa3, A1, A2, A3, Baa1, Baa2, Baa3, Ba1 etc.
- Interest rates are quoted differently depending on the market and might be given as Annualised Percentage Rates (APR's, especially in the bond market) or simple interest rates (money market).
- Debt markets trade on yields and prices based on a number of days per year that could be 360 days a year, 365 days a year, or the actual days in the year, depending on the country and market.

Borrowing: Synonyms

Borrowers receive money at the start. Borrowing is equivalent to selling a debt contract.

To remember that a borrower sells debt, just think about how a borrower receives cash at the **start**. If you're in a shop and you receive cash, that's because you're the seller, not the buying customer.

If a lady sells debt to a man, she receives cash now in return for selling her promise to pay him back the principal and interest later. She sells the debt contract, she is the borrower.

Borrowing is also known as:

- Selling debt (receiving money at the start)
- Having debt liability
- Issuing debt (issuing the debt contract at the start)
- Being short debt (shorting is a synonym of selling)
- Owing money (a borrower 'owes' money)
- Being in debt
- Being a debtor. Or less commonly, a creditee.
- Writing debt (since you write the contractual promise to pay back principal and interest and give the paper contract to the lender)

Lending: Synonyms

Lenders give money at the start. Lending is equivalent to buying a debt contract.

To remember that a lender buys debt, just think about how a lender gives cash at the **start**. If you're in a shop and you pay cash, that's because you're the buying customer, not the selling shop keeper.

If a man buys debt from a lady, he pays cash now to buy her promise to pay him back the principal and interest later. He buys the debt contract, he is the lender.

Lending is also known as:

- Buying debt (giving money at the start)
- Having a debt asset or owning a debt asset
- Investing in debt
- Being long debt (longing is a synonym of buying, shorting is selling)
- Being a creditor. Or less commonly, a debtee.
- Being owed money. Note that this is **not** the past tense of owe. “I’m owed money by you” means the exact opposite of “I owe you money”!

Questions: Debt Terminology

http://www.fightfinance.com/?q=128,129,130,374,234,372,373,

Wholesale Debt Securities

Wholesale debt securities are traded by big financial institutions. They can be classified as being 'money market' or 'bond' securities depending on their original maturity. The Reserve Bank defines bond market debt as having an original maturity of more than one year.

Short term wholesale debt securities

Bills, commonly Bank Accepted Bills (BAB's)

Certificates of Deposit (CD's)

Promissory Notes (PN's)

Long term wholesale debt securities

Bonds, Debentures.

Short Term Wholesale Debt Securities

Short term Wholesale Debt Securities usually have a maturity of less than 1 year when issued. Yields are quoted as simple annual rates, which are mathematically **different** to compound rates such as effective rates and APR's.

Most short term debt securities do not pay coupons and are therefore 'discount securities', which means that their price is less than their face value.

$$Price_{bill} = V_0 = \frac{F_t}{\left(1 + r_{simple} \times \frac{t}{365}\right)}$$

Where F_t is the face value, r_{simple} is the simple annual interest rate and t is days until maturity.

Calculation Example: Bank Accepted Bills (BAB's)

Question: A company issues a bill which is accepted (guaranteed) by a bank. The BAB will mature in 90 days, has a face value of $1 million and an interest rate of 7% pa. What is the price of the bill?

Answer:

$$V_0 = \frac{F_t}{\left(1 + r_{simple} \times \frac{t}{365}\right)}$$

$$= \frac{1{,}000{,}000}{\left(1 + 0.07 \times \frac{90}{365}\right)} = 983{,}032.5882$$

Long-term Wholesale Debt Securities

Usually have an original maturity of more than 1 year. They're commonly known as bonds or notes, same thing these days.

Yields are quoted as APR's, compounding at the same frequency as the coupons are paid. Other names for yield include discount rate, internal rate of return, interest rate or required return.

Most bonds pay coupons. Coupon payments are calculated as face value multiplied by the coupon rate. If the coupons are paid semi-annually, then half of the total coupon is paid every 6 months. Coupon rates are commonly confused with yields, but they are different. Coupon rates are just a convenient way to specify coupon payments.

One confusing thing that doesn't make any sense at all:

- An interest **payment** is a coupon **payment**.
- But an interest **rate** is not a coupon **rate**.

Very odd!

In this course, we will avoid using the words 'interest rate' and 'interest payment'. Instead, use the words 'yield' or 'expected total return' for interest rate and 'coupon or loan payment' for interest payment.

Annualised Percentage Rates (APR's)

Most interest rates are quoted as Annualised Percentage Rates (APR's). This is both by convention and in some countries by law. This is true for credit card rates, mortgage rates, bond yields, and many others. APR's are sometimes called nominal rates, but nominal has another meaning related to inflation so we will avoid calling APR's nominal rates.

The compounding period of an APR is usually not explicitly stated. But usually it can be assumed that **the compounding frequency of an APR is the same as the payment frequency**.

For example, a credit card might advertise an interest rate of 24% pa. This must be an APR since all advertised rates have to be APR's by law. Because credit cards are always paid off monthly, the compounding frequency is per month. Therefore the interest rate is 24% pa given as an APR compounding monthly.

While APR’s are the rate that you see quoted and advertised, unfortunately they **cannot** be used to find present or future values of cash flows. You must convert the APR to an effective rate before doing financial mathematics.

Effective Rates

Effective rates compound only once over their time period, and the time period can be of any length, not necessarily annual.

Effective rates can be used to discount cash flows.

APR's **cannot** be used to discount cash flows, they must be converted to effective rates first.

Note that all of the calculation examples up to here have assumed that the interest rate given is an effective rate.

Calculation Example: Present Values and Effective Rates

Question: What is the present value of receiving \$100 in one year when the effective monthly rate is 1%?

Answer: Since the effective interest rate is per month, the time period must also be in months, so

$$V_o = \frac{C_t}{(1+r)^t}$$

$$= \frac{100}{(1+0.01)^{12}}$$

$$= 88.7449$$

APR's and Effective Rates

- An APR compounding monthly is equal to 12 multiplied by the effective monthly rate.

$$r_{APR\ comp\ monthly} = r_{eff\ monthly} \times 12$$

- An APR compounding semi-annually is equal to 2 multiplied by the effective 6 month rate.

$$r_{APR\ comp\ per\ 6mths} = r_{eff\ 6mth} \times 2$$

- An APR compounding daily is equal to 365 multiplied by the effective daily rate.

$$r_{APR\ comp\ daily} = r_{eff\ daily} \times 365$$

Example: Future Values with APR's

Question: How much will your credit card debt be in 1 year if it's $1,000 now and the interest rate is 24% pa?

Wrong Answer: $V_t = C_0(1 + r)^t = 1000(1 + 0.24)^1 = 1{,}240$

Answer: Since credit cards are paid off per month and rates are by default given as APR’s, the 24% must be an APR compounding monthly. Therefore the effective monthly rate will be the APR divided by 12.

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12} = \frac{0.24}{12} = 0.02$$

$$V_{12mths} = C_0\left(1 + r_{eff\ monthly}\right)^{t_{months}}$$

$$= 1000(1 + 0.02)^{12} = 1{,}268.2418$$

Converting Effective Rates To Different Time Periods

Compounding the rate higher (up to a longer time period):

$$r_{eff\ annual} = \left(1 + r_{eff\ monthly}\right)^{12} - 1$$

$$r_{eff\ semi-annual} = \left(1 + r_{eff\ monthly}\right)^{6} - 1$$

$$r_{eff\ quarterly} = \left(1 + r_{eff\ monthly}\right)^{3} - 1$$

Compounding the rate lower (down to a shorter time period):

$$r_{eff\ monthly} = \left(1 + r_{eff\ annual}\right)^{\frac{1}{12}} - 1$$

$$r_{eff\ daily} = \left(1 + r_{eff\ annual}\right)^{\frac{1}{365}} - 1$$

Calculation Example: Converting Effective Rates

Question: A stock was bought for \$10 and sold for \$15 after 7 months. No dividends were paid. What was the effective annual rate of return?

Answer:

First we find the return over 7 months. This will be the effective 7 month rate of return. Note that the time period is in 7-month blocks:

$$V_0 = \frac{V_t}{(1+r)^t}$$

$$V_0 = \frac{V_{7months}}{\left(1 + r_{eff\ 7month}\right)^{1_{seven\ month\ period}}}$$

$$10 = \frac{15}{\left(1 + r_{eff\ 7month}\right)^{1}}$$

$$\left(1 + r_{eff\ 7month}\right)^{1} = \frac{15}{10}$$

$r_{eff\ 7month} = \frac{15}{10} - 1 = 0.5 = 50\%$, which is the effective 7 month rate.

Now we need to convert the effective 7 month rate to an effective annual rate (EAR). This can be done by 'compounding up' by 12/7 in one step:

$$r_{eff\ annual} = \left(1 + r_{eff\ 7mth}\right)^{\frac{12}{7}} - 1$$

$$= (1 + 0.5)^{12/7} - 1 = 1.0039 = 100.39\%$$

Or it can be broken down into steps:

- Compounding the 7-month rate down to a monthly rate:

$$r_{eff\ monthly} = \left(1 + r_{eff\ 7mth}\right)^{1/7} - 1$$

$$= (1 + 0.5)^{1/7} - 1 = 0.059634 = 5.9634\%$$

- Then compound the monthly rate up to a 12-month (annual) rate:

$$r_{eff\ annual} = \left(1 + r_{eff,monthly}\right)^{12} - 1$$
$$= (1 + 0.059634)^{12} - 1 = 1.0039 = 100.39\%$$

Calculation Example: Converting APR's to Effective Rates

Question: You owe a lot of money on your credit card. Your credit card charges you 9.8% pa, given as an APR compounding per month.

You have the cash to pay it off, but your friend wants to borrow money from you and offers to pay you an interest rate of 10% pa given as an effective annual rate.

Should you use your cash to pay off your credit card or lend it to your friend?

Assume that your friend will definitely pay you back (no credit risk).

Answer:

Since the loan interest rate is an effective rate but the credit card rate is an APR we can't compare them.

Let's convert the credit card's 9.8% APR compounding per month to an effective annual rate:

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12}$$

$$= \frac{0.098}{12} = 0.0081667$$

$$r_{eff\ annual} = \left(1 + r_{eff\ monthly}\right)^{12} - 1$$

$$= (1 + 0.0081667)^{12} - 1 = 0.1025 = 10.25\%$$

So the credit card's 9.8% APR compounding per month converts to an effective annual rate of 10.25%. This is more than the loan's 10% effective annual rate.

You should pay off your credit card which costs 10.25% rather than lend to your friend which only earns 10%, where both rates are effective annual rates.

Questions: APR's and Effective Rates

http://www.fightfinance.com/?q=290,330,16,26,131,49,64,265,

Fully Amortising Loans

Most home loans in Australia are fully amortising. When a borrower of a fully amortising loan makes his last payment, the loan is fully paid off. Sometimes they're called Principal and Interest loans (P&I).

$$V_{0\ fully\ amortising} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

Example: Fully Amortising Loans

Question: Mortgage rates are currently 6% and are not expected to change.

You can afford to pay $2,000 a month on a mortgage.

The mortgage term is 30 years (matures in 30 years).

What is the most that you can borrow using a fully amortising mortgage loan?

Answer: Since the mortgage is fully amortising, at the end of the loan's maturity the whole loan will be paid off.

The bank will lend you the present value of your monthly payments for the next 30 years. This can be calculated using the annuity formula.

The $2,000 payments are monthly, therefore the interest rate and time periods need to be measured in months too.

$$t = 30 \times 12 = 360\ months$$

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12} = \frac{0.06}{12} = 0.005 = 0.5\%$$

$$V_{0\ fully\ amortising} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$= \frac{2000}{0.005}\left(1 - \frac{1}{(1+0.005)^{360}}\right) = \$333{,}583$$

Questions: Fully Amortising Loans

http://www.fightfinance.com/?q=19,87,134,149,172,187,203,204,222,259,

Interest-Only Loans

'Interest only' means that all of your loan payments go towards the interest, so the principal is not paid off until the very end of the mortgage. Usually this big amount is paid off by re-financing which is borrowing again, or by selling the house.

$$V_{0\ interest\ only} = \frac{C_1}{r} = \frac{C_{1\ monthly}}{r_{eff\ monthly}} = \frac{C_{1\ monthly}}{\left(\frac{r_{APR\ comp\ monthly}}{12}\right)}$$

$V_0 = \frac{C_1}{r}$ is the 'perpetuity without growth' formula. This makes sense since an interest-only loan that is repeatedly refinanced always has the same principal, it's never paid off. If the interest rate is constant forever, then the borrower will pay constant interest perpetually.

Another way of looking at the interest-only loan is to assume that the final principal payment is paid off without refinancing. The initial price (V_0) is the present value of the interest payments ($C_1, C_2, \ldots$) and the final principal (V_T):

$$V_0 = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{V_T}{(1+r)^T}$$

$$\boldsymbol{V_0} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{\boldsymbol{V_0}}{(1+r)^T}\,, \qquad \text{Since } \mathrm{V_0} = \mathrm{V_T}$$

$$V_0 - \frac{V_0}{(1+r)^T} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$V_0\left(1 - \frac{1}{(1+r)^T}\right) = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$V_0 = \frac{C_1}{r}$$

Again, the interest-only loan with constant interest rates can be valued as a perpetuity without growth.

Calculation Example: Interest Only Loans

Question: Using the same data as before, what is the most that you can borrow using an interest only mortgage loan? Again, assume constant interest rates.

Answer: Interest-only loans are equivalent to perpetuities with no growth.

$$V_{0\ interest\ only} = \frac{C_1}{r} = \frac{C_{1\ monthly}}{r_{eff\ monthly}} = \frac{C_{1\ monthly}}{\left(\frac{r_{APR\ comp\ monthly}}{12}\right)}$$

$$= \frac{2000}{(0.06/12)} = \$400{,}000$$

Questions: Interest Only Loans

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Bond Pricing exactly one period before the next coupon

The price of a bond is the present value of the coupons and the principal. The coupons are an annuity and the principal (or face value) is a single payment, therefore:

$$P_{0,bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face_T}{\left(1 + r_{eff}\right)^T}$$

Note that this equation assumes that the next coupon is paid in exactly one period. This will be the case for a bond that was just issued or a bond that just paid a coupon.

Care must be taken when choosing r_{eff} and T so that they are consistent with the time period between coupon payments (C_1).

Remember that bond yields are given as APR's compounding at the same frequency as coupons are paid, so they normally have to be converted to effective rates before using them in the above equation.

Bond Pricing Conventions

US and Australian fixed coupon bonds generally pay semi-annual coupons. Therefore the yields are quoted as APR's compounding semi-annually.

European bonds generally pay annual coupons. Therefore the yields are quoted as APR's compounding annually, which is the same thing as an effective annual rate.

Calculation Example: Bond Pricing exactly one period before the next coupon

Question: An Australian company issues a fixed-coupon bond. The bond will mature in 3 years, has a face value of $1,000 and a coupon rate of 8%. Yields are currently 5% pa. What is the price of the bond?

Answer: Since it is an Australian bond, we assume that it pays semi-annual coupons as is customary.

Therefore each 6 month coupon will be:

$$C_{semi-annual} = Face\ value \times \frac{coupon\ rate}{2} = 1{,}000 \times \frac{0.08}{2}$$

$$= \$40$$

The number of time periods T must be consistent with the coupon payment frequency of 6 month periods, so

$$T = 3 \text{ years } \times 2$$

$$= 6 \text{ semi-annual periods}$$

The yield of 5% can be assumed to be an APR compounding every 6 months, the same frequency as the coupon payments. We need to find the effective 6 month rate to discount the 6-month coupons, so:

$$r_{eff\ 6month} = r_{APR\ comp\ semi\ annually} \div 2$$

$$= 0.05 \div 2 = 0.025$$

To find the bond price,

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face_T}{\left(1 + r_{eff}\right)^T}$$

$$= \frac{1000 \times 0.08/2}{0.05/2}\left(1 - \frac{1}{(1 + 0.05/2)^{3\times2}}\right) + \frac{1000}{(1 + 0.05/2)^{3\times2}}$$

$$= \frac{40}{0.025}\left(1 - \frac{1}{(1 + 0.025)^6}\right) + \frac{1{,}000}{(1 + 0.025)^6}$$

$$= 220.3250145 \quad + \quad 862.296866$$

$$= \$1{,}082.62$$

Bond Yields and Coupon Rates - Important

Here are three similar bonds which differ only in their coupon rate.

Bond	Face value	Maturity	Current yield pa	Coupon rate pa	Price*	**Bond type**
1	$100	3 yrs	5%	0%	$86.23	**Discount**
2	$100	3 yrs	5%	5%	$100.00	**Par**
3	$100	3 yrs	5%	10%	$113.77	**Premium**

* Coupons are assumed to be paid semi-annually.

- Bonds issued at 'par' will have:
  - A price equal to face value
  - A yield equal to the coupon rate.
- Zero coupon bonds are discount bonds.

Calculation Example: Bonds issued at par

Question: An Australian company issues a bond at **par**. The bond will mature in 3 years, has a face value of $1,000 and a coupon rate of 8%. What is the price of the bond?

Answer: This is a trick question, no calculations are required. Since the bond was issued at par, the price must be equal to the face value. Therefore the price is $1,000. Current yields in the bond market must also be equal to 8% pa, the same as the bond's coupon rate.

Calculation Example: Zero coupon bonds

Question: An Australian company issues a zero coupon bond. The bond will mature in 3 years and has a face value of $1,000. If the current price of the bond is $700, what is the current yield on the bond, given as an APR compounding semi-annually?

Answer: Zero coupon bonds pay no coupons. Therefore the price of the bond is just the present value of the principal.

To find current yields we need to solve for the discount rate:

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= 0 + \frac{Face}{\left(1 + r_{eff}\right)^{T}}$$

$$700 = \frac{1{,}000}{\left(1 + r_{eff\ 6mth}\right)^6}$$

$$700 \times \left(1 + r_{eff\ 6mth}\right)^6 = 1{,}000$$

$$\left(1 + r_{eff\ 6mth}\right)^6 = \frac{1{,}000}{700}$$

$$1 + r_{eff\ 6mth} = \left(\frac{1{,}000}{700}\right)^{\frac{1}{6}}$$

$$r_{eff\ 6mth} = \left(\frac{1{,}000}{700}\right)^{\frac{1}{6}} - 1$$

$$= 0.061248265$$

We need to convert this rate to an APR compounding every 6 months since that is how bond yields are quoted in Australia.

$$r_{APR\ comp\ per\ 6mths} = r_{eff\ 6mth} \times 2$$

$$= 0.061248265 \times 2$$

$$= 0.12249653 = 12.249653\%$$

For the exam, note that you will not be asked to find the yield on coupon-paying bonds since that requires trial-and-error or a computer that can do it for you (use a spreadsheet's IRR or YIELD formula).

But you may be asked to find the yield on a simple zero coupon bond like we did in this question.

Note that instead of finding the effective semi-annual rate and converting to an APR compounding semi-annually at the end, the bond pricing equation can be set up so that the APR compounding semi-annually is calculated from the start:

$$700 = \frac{1{,}000}{\left(1 + \frac{r_{APR\ comp\ 6mth}}{2}\right)^6}$$

$$r_{APR\ comp\ 6mth} = 2\left(\left(\frac{1{,}000}{700}\right)^{\frac{1}{6}} - 1\right)$$

$$= 2 \times 0.061248265$$

$$= 0.12249653$$

$$= 12.249653\%$$

Bond Pricing in between coupons

To price a bond in between coupon periods, the bond price must be grown by the total required return, r_{eff}. This reflects the increase in the bond price as the next coupon approaches.

$$P_{0,bond} = \left(\frac{C_{1-F}}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^{T}}\right) + \frac{Face_{T-F}}{\left(1 + r_{eff}\right)^{T}}\right)\left(\mathbf{1} + \boldsymbol{r}_{\boldsymbol{eff}}\right)^{\boldsymbol{F}}$$

Where:

F is the fraction of a coupon period that has elapsed since the last coupon.

T is the number of coupons to be paid in the future.

C is the next coupon, which will be paid in $(1 - F)$ coupon periods from now (t=0).

$Face$ is the face value of the bond, which will be paid in T-F coupon periods from now.

Calculation Example: Bond pricing in between coupons

Question: A 10 year government bond paying 3% pa semi-annual coupons with a face value of $100 was issued 8 months ago on 15 December 2014 at a yield of 3%.

Today is 15 August 2015 and yields are now 2.8%. What is the current price of the bond?

Ignore the actual number of days in each month and assume that every month is 1/12 of a year, so the bond was issued 8 months ago from today, 15 August 2015.

Answer: The bond's last coupon was on 15 June 2015, 2 months ago. This is 2/6 of a semi-annual coupon period so F=2/6. There are only 19 semi-annual coupons left, since one was already paid on 15 June 2015.

$$P_{0,bond} = \left(\frac{C_{1-F}}{r_{eff}}\left(1-\frac{1}{\left(1+r_{eff}\right)^{T}}\right)+\frac{Face_{T-F}}{\left(1+r_{eff}\right)^{T}}\right)\left(\mathbf{1}+\boldsymbol{r}_{\boldsymbol{eff}}\right)^{\boldsymbol{F}}$$

$$= \left(\frac{0.03\times 100/2}{0.028/2}\left(1-\frac{1}{(1+0.028/2)^{19}}\right)+\frac{100}{(1+0.028/2)^{19}}\right)(\mathbf{1}+\mathbf{0.028/2})^{\mathbf{2/6}}$$

$$= (24.87274706 + 76.78543608)(1+0.028/2)^{2/6}$$

$$= 101.6581831 \times (1+0.028/2)^{2/6} = 102.1303912$$

Questions: Bond Pricing

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Term Structure of Interest Rates

Long term interest rates are based on expectations of future short term interest rates.

We will discuss spot and forward rates, yield curves, and then two important theories of interest rates:

- Expectations hypothesis
- Liquidity premium theory

Spot and Forward Interest Rates

Spot rate: An interest rate measured from now until a future time. For example, a 3-year zero-coupon bond with a yield of 8% pa has a 3-year pa spot rate of $r_{0-3,yearly} = 0.08$ pa. Note that spot rates can be from now until ***any*** future time.

Forward rate: An interest rate measured from a future time until a more distant future time. For example, if a company promised, **one year from now,** to issue a 3-year zero-coupon bond with a yield of 8% pa, then the forward rate from years 1 to 4 would be $r_{1-4,yearly} = 0.08$. Forward rates are sometimes written with an 'f' rather than 'r'.

Spot and forward rates can be quoted as APR's or effective rates.

Term Structure of Interest Rates: The Expectations Hypothesis

Expectations hypothesis is that long term spot rates (plus one) are the geometric average of the shorter term spot and forward rates (plus one) over the same time period. Mathematically:

$$1 + r_{0\to T} = \left((1 + r_{0\to 1})(1 + r_{1\to 2})(1 + r_{2\to 3})\ldots\left(1 + r_{(T-1)\to T}\right)\right)^{\frac{1}{T}}$$

or

$$(1 + r_{0\to T})^{T} = (1 + r_{0\to 1})(1 + r_{1\to 2})(1 + r_{2\to 3})\ldots(1 + r_{(T-1)\to T})$$

Where T is the number of periods and all rates are effective rates over each period.

Calculation Example: Term Structure of Interest Rates

Question: The following US Government Bond yields were quoted on 5/3/2012 (sourced from Bloomberg):

- 6-month zero-coupon bonds yielded 0.11%.
- 12-month zero-coupon bonds yielded 0.16%.

Find the forward rate from month 6 to 12. Quote your answer as a yield in the same form as the above yields are quoted.

Remember that US (and Australian) bonds normally pay semi-annual coupons.

Answer: Even though these are zero-coupon bonds, since they are US bonds the yield would be quoted as an APR compounding semi-annually since all coupon bonds pay semi-annual coupons. This means that our answer should be quoted in the same form, as an APR compounding semi-annually.

Therefore we have to convert the APR compounding every 6 months to an effective 6 month yield by dividing it by 2.

$$r_{0\to 0.5yr,eff\ 6mth} = \frac{r_{0\to 0.5yr,APR\ comp\ semi-annually}}{2}$$

$$= \frac{0.0011}{2} = 0.00055$$

$$r_{0\to 1yr,eff\ 6mth} = \frac{r_{0\to 1yr,APR\ comp\ semi-annually}}{2}$$

$$= \frac{0.0016}{2} = 0.0008$$

We want to find $r_{0.5yr\to 1yr,eff\ 6mth}$, which is the effective 6 month forward rate over the second 6 month period (0.5 years to 1 year).

Applying the term structure of interest rates equation:

$$(1 + r_{0\to T})^T = (1 + r_{0\to 1})(1 + r_{1\to 2})(1 + r_{2\to 3}) \dots (1 + r_{(T-1)\to T})$$

$$\left(1 + r_{0\to 1yr,eff\ 6mth}\right)^2 = \left(1 + r_{0\to 0.5yr,eff\ 6mth}\right)\left(1 + r_{0.5\to 1yr,eff\ 6mth}\right)$$

$$(1 + 0.0008)^2 = (1 + 0.00055)\left(1 + r_{0.5\to 1yr,eff\ 6mth}\right)$$

$$r_{0.5\to 1yr,eff\ 6mth} = \frac{(1+0.0008)^2}{(1+0.00055)} - 1$$

$$= 0.001050062$$

But this is an effective 6 month rate. Let's convert it to an APR compounding every 6 months.

$$r_{0.5\to 1yr,APR\ comp\ 6mths} = r_{0.5\to 1yr,eff\ 6mth} \times 2$$

$$= 0.001050062 \times 2$$

$$= 0.002100124 = 0.21\% pa$$

Note that this forward rate APR from 0.5 years to 1 year is bigger than both of the bond yield APR's (which are spot rates). This makes sense since we have a normal upward sloping yield curve ($r_{0\to 0.5yr} < r_{0\to 1yr}$) so the forward rate

($r_{0.5\to 1yr}$) should be greater than the spot rates ($r_{0\to 0.5yr}$ *and* $r_{0\to 1yr}$).

Quick method: Convert rates in the term structure equation

Interest rate conversion from anualised percentage rates (APR's) to effective returns can be a headache. Most people prefer to do the APR to effective rate conversion in the expectations formula itself:

$$(1 + r_{0\to T\ eff})^{T} = (1 + r_{0\to 1\ eff})(1 + r_{1\to 2\ eff}) \ldots (1 + r_{(T-1)\to T\ eff})$$

$$(1 + r_{0\to 1yr,eff\ 6mth})^{2} = (1 + r_{0\to 0.5yr,eff\ 6mth})(1 + r_{0.5\to 1yr,eff\ 6mth})$$

$$\left(1 + \frac{r_{0\to 1yr,APR\ 6mths}}{2}\right)^{2} = \left(1 + \frac{r_{0\to 0.5yr,APR\ 6mths}}{2}\right)\left(1 + \frac{r_{0.5\to 1yr,APR\ 6mths}}{2}\right)$$

$$\left(1 + \frac{0.0016}{2}\right)^{2} = \left(1 + \frac{0.0011}{2}\right)\left(1 + \frac{r_{0.5\to 1yr,APR\ 6mths}}{2}\right)$$

$$r_{0.5\to 1yr,APR\ 6mths} = \left(\frac{\left(1+\frac{0.0016}{2}\right)^2}{\left(1+\frac{0.0011}{2}\right)} - 1 \right) \times 2 = 0.002100124$$

This forward rate from 6 months to one year is 0.21% pa given as an APR compounding every 6 months.

Yield Curves

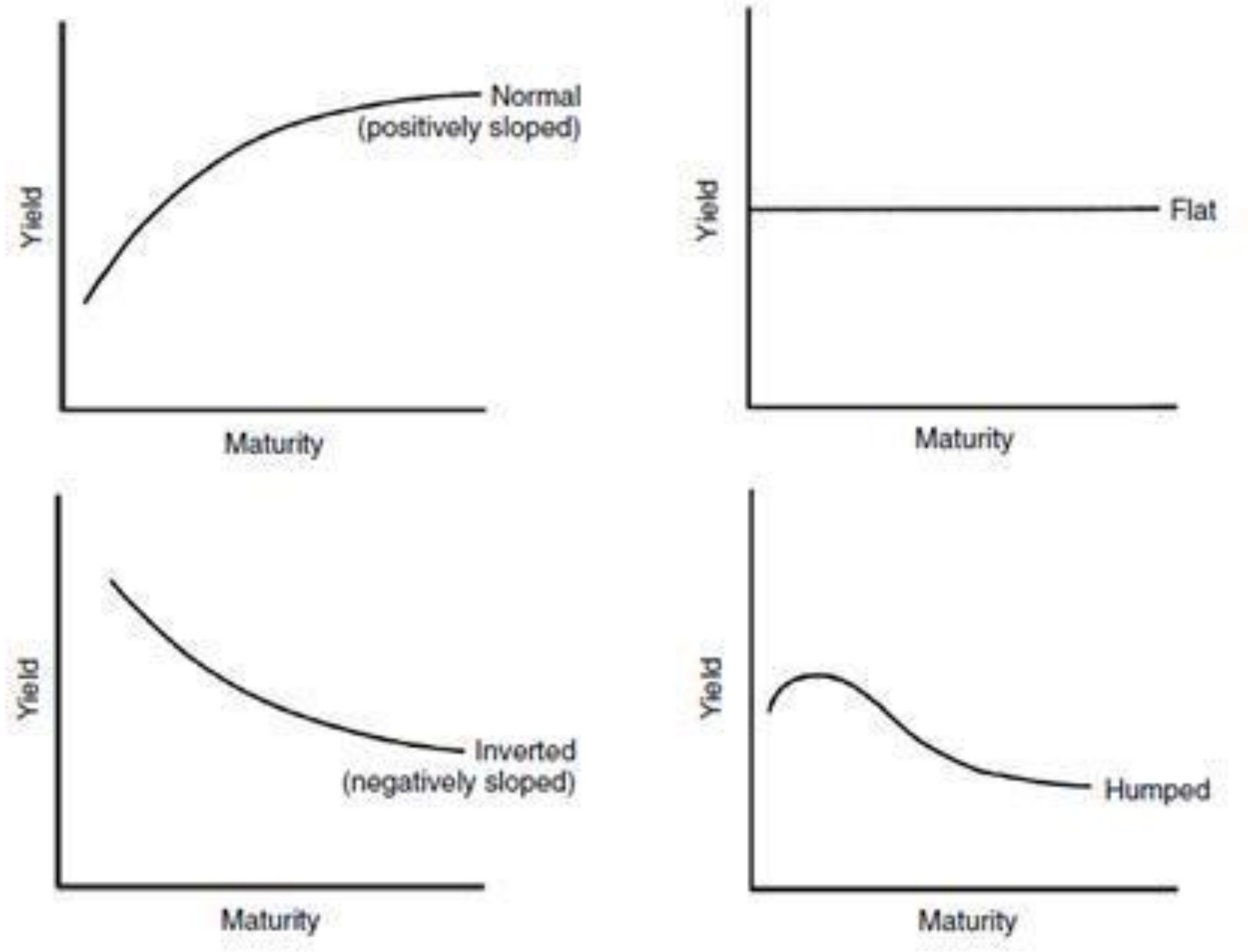
Yield curves show the behaviour of short and long-term interest rates and can give an indication of expected future interest rates.

Yield curves can be flat, normal, inverse, or some combination. The x-axis of a yield curve is the time to maturity of the bond, and the y-axis is the yield of the bond.

A **flat** yield curve is a straight horizontal line. Short and long term spot rates are equal, and yearly spot and forward rates are also equal. Other names for flat interest rates are 'constant', 'unchanging', or 'level' interest rates.

A **normal** yield curve is an upward sloping line or curve. Short term spot rates are less than long term spot rates. Yearly spot rates are less than yearly forward rates. Other names for normal yield-curves are 'upward sloping' or 'steep ' yield curves. These yield curves are 'normal' since yields usually exhibit this behaviour.

An **inverse** yield curve is a downward sloping line or curve. Short term spot rates are more than long term spot rates. Yearly spot rates are more than yearly forward rates. Other names for inverse yield-curves are 'downward sloping' or 'inverted ' yield curves.


Normal
(positively sloped)
Yield
Maturity
Flat
Yield
Maturity
Inverted
(negatively sloped)
Yield
Maturity
Humped
Yield
Maturity

An Extension: Liquidity Premium Theory

The expectations hypothesis assumes that investors are indifferent between investing in a 10 year bond, or investing in a one year bond, then investing in another 1 year bond after the first is repaid, and so on for 10 years.

Most investors would prefer to lend lots of short term bonds repeatedly rather than one big long one. The reason is that the long-term bond locks up the investor's cash and she loses the option to change her mind and do something else with the cash.

The liquidity premium theory suggests investors are only enticed to lend their cash out long-term if they are rewarded for doing so in the form of higher long-term rates compared to short term rates. This means that forward rates will be higher than the expected spot rates over the same time period.

For example, if the forward rate from years 1 to 2 is 8% now, then 1 year later the spot rate (from years 0 to 1) would tend to be less, say 7.5%.

This theory explains why the up-ward sloping yield curve is normal, since spot rates would tend to be less than forward rates.

Real World Example: Yield Curves and Term Structure of Interest Rates

See the below sources for an interesting view of yield curves and the term structure of interest rates.

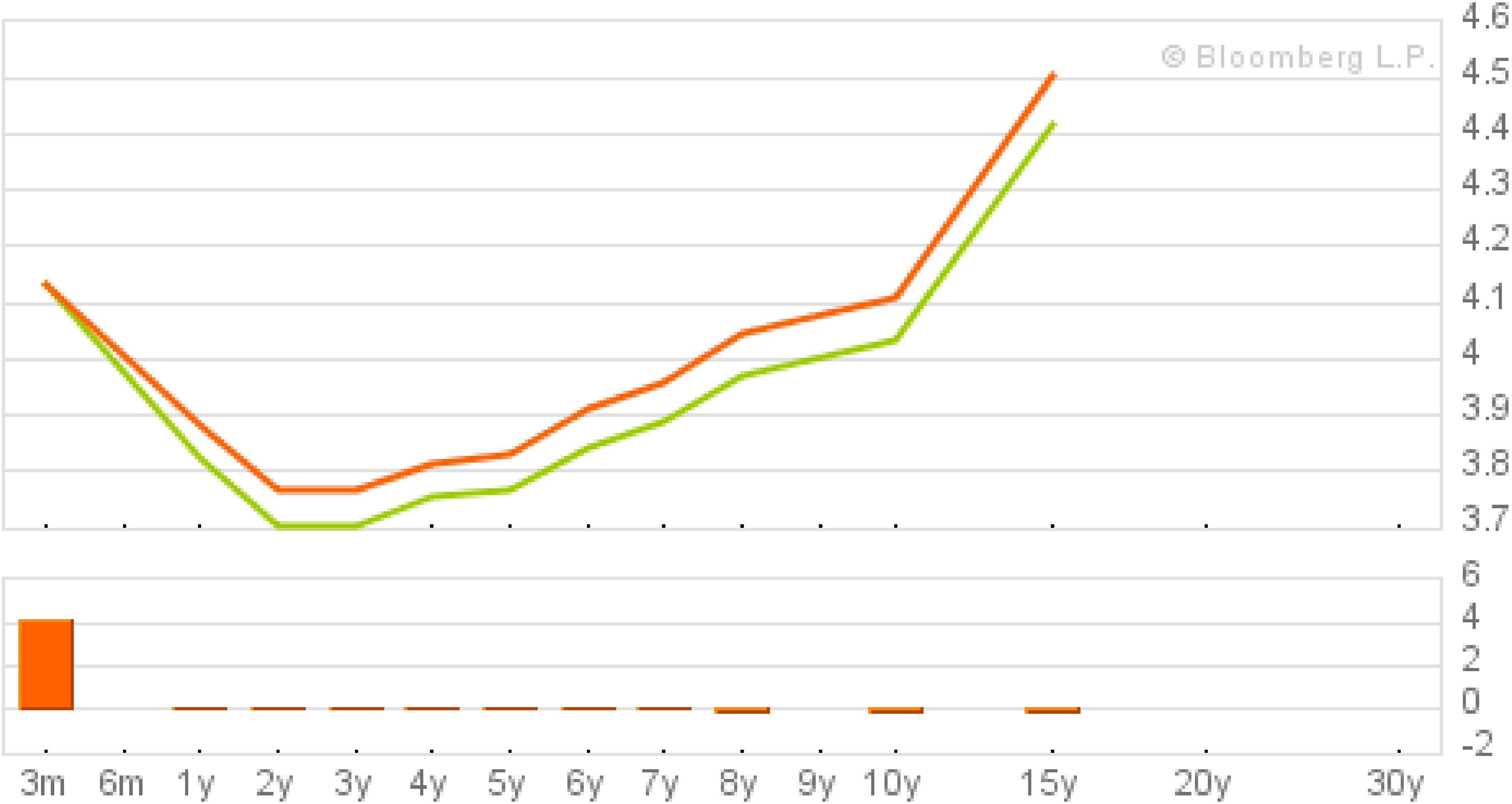
Australian Federal Government (Fixed Coupon) Bond Yields:

http://www.bloomberg.com/markets/rates-bonds/government-bonds/australia

Table of yields on evening of 5/3/2012. Source: Bloomberg.

Australian Government Bonds	COUPON	MATURITY	PRICE/YIELD	PRICE/YIELD CHANGE	TIME
3-Month	0.000	06/08/2012	4.15 / 4.15	98.943 / 4.150	02/24
1-Year	6.500	05/15/2013	103.06 / 3.83	0.058 / -0.055	00:39
2-Year	6.250	06/15/2014	105.48 / 3.71	0.131 / -0.061	00:39
3-Year	6.250	04/15/2015	107.39 / 3.71	0.181 / -0.062	00:39
4-Year	4.750	06/15/2016	103.87 / 3.76	0.237 / -0.060	00:39
5-Year	6.000	02/15/2017	109.95 / 3.77	0.284 / -0.061	00:39
6-Year	5.500	01/21/2018	108.60 / 3.85	0.374 / -0.069	00:39
7-Year	5.250	03/15/2019	108.25 / 3.90	0.452 / -0.071	00:39
8-Year	4.500	04/15/2020	103.63 / 3.97	0.537 / -0.077	00:39
10-Year	5.750	05/15/2021	113.05 / 4.04	0.649 / -0.080	00:39
15-Year	4.750	04/21/2027	103.53 / 4.43	0.931 / -0.084	00:38

Orange line: current yield, Green line: previous close (yesterday's) yield. As at 5/3/2012. Note the humped curve.



Interest Rate Futures to Predict Interest Rates

The Australian Securities Exchange (ASX) trades many derivatives including interest rate futures.

Futures are similar to forward contracts in that they lock in the price and terms of trading a security now, but the trade takes place in the future. Unlike forward contracts, futures are exchange-traded rather than OTC, and they require margin deposits by the long and short traders which reduces credit risk.

Interest rate futures bind the buyer and seller to trade an interest rate security (debt instrument) at a set price in the future.

30-day Interbank Cash Rate Futures

One of the most interesting futures contracts is the 30-day Interbank Cash Rate Futures Contract.

It can be used to predict the Reserve Bank of Australia (RBA) overnight cash rate, also called the target rate, policy rate or inter-bank overnight money-market rate.

The ASX publishes the predicted market-implied probability of RBA target rate changes on its RBA Rate Indicator web site:

http://www.asx.com.au/prices/targetratetracker.htm

ASX's Target Rate Tracker as at 5/3/2012, the day before the Tuesday 6 March 2012 RBA board meeting.

The table below highlights how market expectations of the **probability** of cash rate decrease at the next RBA Board meeting has evolved in recent days. Since 7/12/2011 the RBA cash rate has been 4.25%.

Trading Day	No Change	Decrease to 4.00%
23 February	75%	25%
24 February	80%	20%
27 February	80%	20%
28 February	80%	20%
29 February	80%	20%
1 March	85%	15%
2 March	85%	15%
5 March	88%	12%