Yield to Maturity (YTM)

The yield to maturity (YTM) is the promised return that discounts a bond's coupons and face value to the present, which equals the bond price. The YTM in the equation below is represented as an effective rate r_{eff}. If the coupons are annual then it will be an effective annual rate, if the coupons are semi-annual then it will be an effective 6 month rate.

$$P_{0,bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face_T}{\left(1 + r_{eff}\right)^T}$$

The YTM is the promised internal rate of return (IRR) of the bond.

YTM Discounts Cash Flows at Different Times

Since the YTM discounts all coupons and the face value which are at different times, it's sort of a mutant Frankenstein rate.

Two bonds with the same maturity, risk and face value but different coupons will often have different YTM's when the yield curve is not flat.

Say a 3 year annual coupon government bond with a face value of $100 and coupon rate of 5% pa has a YTM of 4%. The yield curve is normal or upward sloping.

If you were asked to find the present value of a risk-free \$200 payment in 3 years you **cannot** simply discount the \$200 by 4% like so: $200/(1 + 0.04)^3$.

This is because the required return for a payment in 3 years is not the 4% YTM of the bond. The YTM is a sort of blend of the yields at every coupon payment and the face value. If the yield curve it upward sloping, the appropriate 3 year required return or yield would be slightly higher than the YTM of 4%.

To properly discount a payment in 3 years, you need the 3 year **zero coupon yield**.

Zero Coupon Rate

Zero coupon rates are the yield to maturities (YTM's) of bonds that pay no coupons. These rates are more useful than coupon-paying bonds' YTM's since they can be used to precisely discount a payment at a specific time.

The zero coupon yield is the promised return that discounts a bond's face value only to the present, which equals the zero coupon bond's price. Zero coupon yields are not polluted by coupon payments.

$$P_{0,zero\ coupon\ bond} = PV(principal)$$

$$= \frac{Face_T}{\left(1 + z_{eff}\right)^T}$$

Say a 3 year annual zero-coupon government bond with a face value of $100 has a YTM (which is its zero-coupon yield) of 4.5%. The yield curve is normal or upward sloping.

If you were asked to find the present value of a risk-free $200 payment in 3 years you **can** simply discount the $200 by 4.5% like so: $200/(1+0.045)^3$, provided the 4.5% zero coupon yield is given as an effective annual rate.

Boot Strapping to Find the Zero Coupon Yield Curve

Boot strapping is a two-step process that can be used to find the zero-coupon yields starting from the short term spot zero rates to the longer term spot rates.

It's called boot-strapping because when you do up your boot straps or laces, you start at the bottom near your toe and work higher and higher. The zero coupon yields must be built up to longer and longer zero rates, there's no short cut.

Boot Strapping: A 2-Step Process

Boot strapping is a repetitive two-step process:

1. Price the bond using the YTM. For a one year semi-annual coupon bond with YTM $r_{0\to1}$:

$$P_{0,1yr\ bond} = \frac{C_{0.5}}{(1 + r_{0\to1}/2)^{0.5\times2}} + \frac{C_1 + F_1}{(1 + r_{0\to1}/2)^{1\times2}}$$

2. Use that price ($P_{0,1yr\ bond}$) to solve for the longer-term zero-coupon yield ($z_{0\to1}$) that discounts the last payment ($C_1 + F_1$) given that you already know all shorter-term zero coupon yields ($z_{0\to0.5}$):

$$P_{0,1yr\ bond} = \frac{C_{0.5}}{(1 + z_{0\to0.5}/2)^{0.5\times2}} + \frac{C_1 + F_1}{(1 + \boldsymbol{z_{0\to1}}/2)^{1\times2}}$$

Repeat!

Note that the very first zero coupon yield ($z_{0\to0.5}$ in the example) requires knowledge of the YTM ($r_{0\to0.5}$) of a short term bond (0.5 yr bond) which pays no coupons or where the coupon is at the same time as the principal, because then the YTM will also be the zero coupon yield ($r_{0\to0.5} = z_{0\to0.5}$). This is because there's only a single cash flow at maturity:

$$P_{0,0.5yr\ bond} = \frac{C_{0.5} + F_{0.5}}{(1 + \boldsymbol{r_{0\to0.5}}/2)^{0.5\times2}} = \frac{C_{0.5} + F_{0.5}}{(1 + \boldsymbol{z_{0\to0.5}}/2)^{0.5\times2}}$$

So this very first short term bond requires no 2-step process, the YTM is always the zero coupon yield.