***Calculation Example: No-arbitrage Valuation***

A stock is priced at $50. In one year the stock price will increase to $60 or fall to $45. The continuously compounded risk free rate is 10% p.a.. Value a European call option that expires in one year with a strike price of $52.

1. Make a portfolio at $t=0$ of short 1 option and long $Δ$ shares.

$$V\_{0}=-f\_{0}+Δ.S\_{0}$$

$$ =-f\_{0}+Δ×50$$

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1. Make two expressions for portfolio value at $t=1$ for when the share price goes up and down.

$$V\_{1u}=-f\_{1u}+Δ.S\_{1u}$$

$$ =-max(S\_{1u}-K\_{1}, 0)+Δ.S\_{1u}$$

$$ =-max(60-52, 0)+Δ×60$$

$$ =-8+Δ×60$$

$$V\_{1d}=-f\_{1d}+Δ.S\_{1d}$$

$$ =-max(S\_{1d}-K\_{1}, 0)+Δ.S\_{1d}$$

$$ =-max(50-52, 0)+Δ×45$$

$$ =-0+Δ×45$$

1. Set these expressions equal to each other and solve for $Δ$ (Delta).

$$ V\_{1u} = V\_{1d}$$

$$-f\_{1u}+Δ.S\_{1u}=-f\_{1d}+Δ.S\_{1d}$$

$$-8+Δ×60=-0+Δ×45$$

$$Δ=\frac{8}{15}=0.533$$

1. So if we are long 0.533 of a share and short 1 call option, this portfolio will be worth the same amount whether the share price rises or falls. Since the portfolio is worth the same amount either way, there is no risk. Therefore we can discount the value of the portfolio by the risk free rate. Now solve for $f\_{0}$ the value of the option.

$$ V\_{0} = V\_{1u}.e^{-r.t}$$

$$-f\_{0}+Δ.S\_{0}=\left(-f\_{1u}+Δ.S\_{1u}\right).e^{-r.t}$$

$-f\_{0}+\frac{8}{15}×50=\left(-8+\frac{8}{15}×60\right).e^{-0.1×1}$

$-f\_{0}+26.667=21.716$

$$f\_{0}= 4.95$$

So the value of the call option now, which is the same as its price or premium right now, is $4.95.