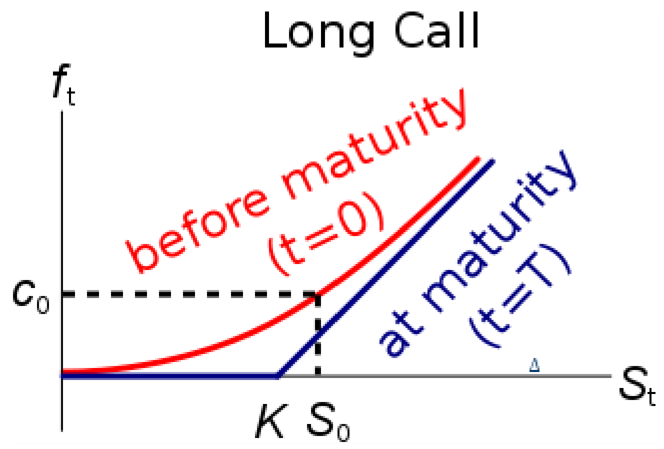
European Option Valuation Without Dividends

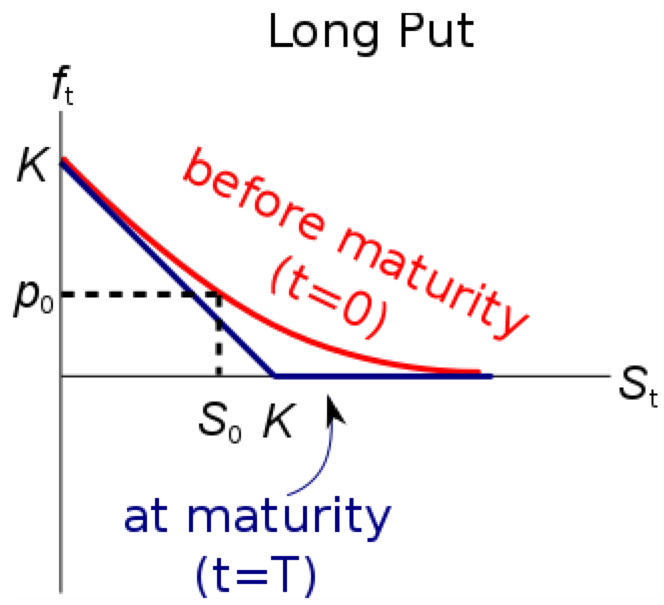
$$c_0 = \quad S_0.N[d_1] \quad - K_T.e^{-rT}.N[d_2]$$

$$p_0 = -S_0.N[-d_1] + K_T.e^{-rT}.N[-d_2]$$

$$d_1 = \frac{\ln[S_0/K_T] + \left(r + \frac{\sigma^2}{2}\right).T}{\sigma.\sqrt{T}}$$

$$d_2 = \frac{\ln[S_0/K_T] + \left(r - \frac{\sigma^2}{2}\right).T}{\sigma.\sqrt{T}} = d_1 - \sigma.\sqrt{T}$$



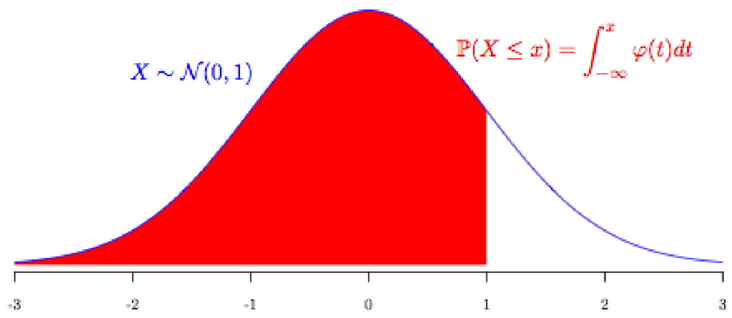


$N[Z]$

$N[Z]$ gives the probability that the standardised normally distributed random variable x with mean 0 and variance 1 is less than Z:

$$N[Z] = \Pr(x < Z)$$

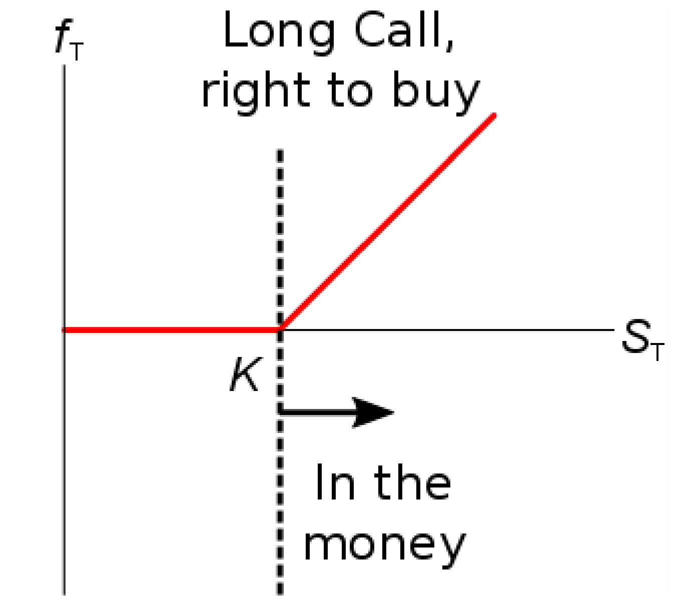
It's normally looked up on the standard normal Z table. It can also be calculated in MS Excel using the '=normsdist()' function.



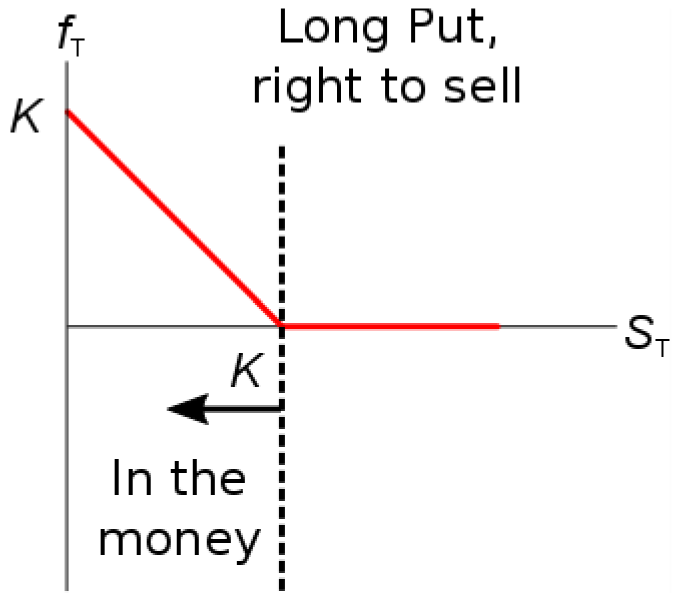
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

$N[d_2]$ *Probability*

$N[d_2] = \Pr(S_T > K_T)$ for a European call option, which is the probability that the call option ends up being in the money at maturity in a risk-neutral world where all assets earn the risk free rate r.



$N[-d_2] = \Pr(S_T < K_T)$ for a European put option, which is the probability that the put option ends up being in the money at maturity in a risk-neutral world.



To get the risk-averse probability in the real world, replace r in $N[d_2]$ with the underlying asset's forecast natural logarithm of arithmetic average gross discrete returns (LAAGDR):

$$LAAGDR_{0\to T} = \ln\left(\frac{1}{T}.\sum_{t=1}^{T}\left(\frac{p_t}{p_{t-1}}\right)\right)$$

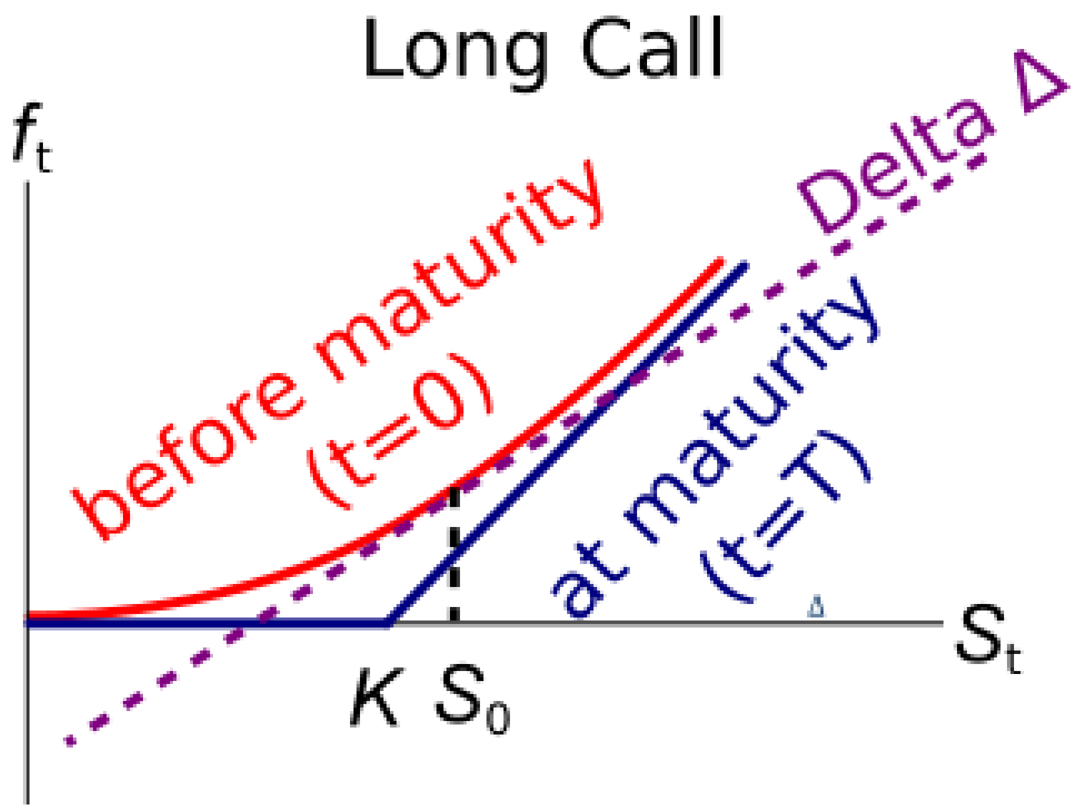
See the lecture on return distributions for more detail.

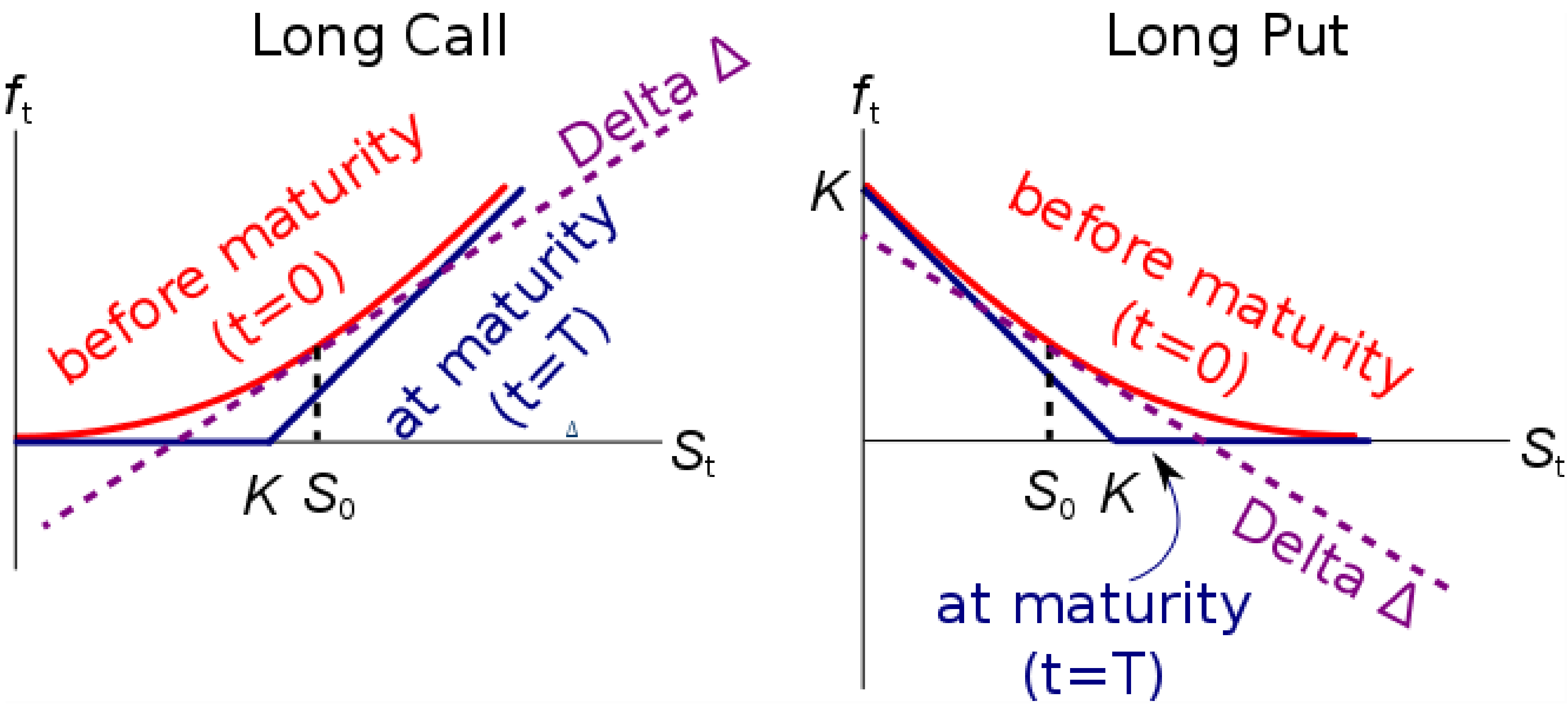
$N[d_1]$ *Option Delta* Δ

$N[d_1] = \Delta$, the European call option Delta at the current underlying asset price S_0 which has T time until maturity.

The Delta is a tangential gradient, it's the increase (rise) in option price f_t divided by the increase (run) of the underlying asset price S_t.

For every 1 cent increase in the underlying asset price, the option price will increase by Delta cents.




Long Call
f_t
before maturity
(t=0)
Delta Δ
at maturity
(t=T)
K S_0
S_t
Long Put
f_t
K
before maturity
(t=0)
S_0 K
S_t
Delta Δ
at maturity
(t=T)

Portfolio Delta Δ_p

If a portfolio's Delta is zero then a small change in the underlying asset price will lead to no change in the value of the portfolio, so it's hedged. The Delta of a portfolio is simply the sum of the assets' Deltas. Let n_1 be the number of asset 1's, and let m be the total number of assets. Then the portfolio Delta (Δ_p) is:

$$\Delta_p = \Delta_1 . n_1 + \Delta_2 . n_2 + \cdots + \Delta_m . n_m$$

Relationship to the No-arbitrage Binominal Option pricing Delta

This Black-Scholes-Merton Delta ($\Delta = N[d_1]$) is the same Delta as the No-arbitrage Binominal Option pricing formula's delta:

$V_0 = -f_0 + \Delta. S_0$