Question 1: Which of the following derivative instruments has a zero value when it's first agreed to?

*(a) Long futures contract.

(b) Out-of-the money European-style long call option.

(c) At-the-money European-style long call option.

(d) In-the-money European-style long call option.

(e) At-the-money American-style long put option.

Question 2: Which of the following derivative instrument positions require an initial deposit into a margin account?

*(a) Long futures contract.

(b) Out-of-the money European-style long call option.

(c) At-the-money European-style long call option.

(d) In-the-money European-style long call option.

(e) At-the-money American-style long put option.

Question 3: Which of the following derivative instrument positions could you exercise before maturity?

(a) Long futures contract.

(b) Out-of-the money European-style long call option.

(c) At-the-money European-style long call option.

(d) In-the-money European-style long call option.

*(e) At-the-money American-style long put option.

Question 4: Which of the following types of call or put options **CANNOT** be exactly priced using the Black-Scholes equation?

(a) European-style call or put options on non-dividend-paying paying stocks.

(b) European-style call or put options on dividend-paying paying stocks.

(c) American-style call or put options on non-dividend-paying paying stocks.

*(d) American-style call or put options on dividend-paying paying stocks.

(e) The Black-Scholes equation gives an exact price for all of the above types of options.

Question 1 (total of 8 marks): A stock index is expected to pay a continuously compounded dividend yield **4**% pa for the foreseeable future. The index is currently at **5,000** points, the continuously compounded total required return is **9**% p.a and its standard deviation of returns is **30**% p.a.. An investor has just taken a long position in an **8**-month **call** option contract on the index with a strike price of **5,100**. Compute the call option price in index points using the Black-Scholes model.

Question 1a (3 marks): Calculate d_1.

*d1= 0.177713363

Question 1b (1 mark): Calculate d_2.

*d2= -0.067235611

Question 1c (1 mark): Calculate $N(d_1)$ using the tables in the back of this exam paper.

*N(d1)= 0.570525955

Question 1d (1 mark): Calculate $N(d_2)$ using the tables in the back of this exam paper.

*N(d2)= 0.473197068

Question 1e (2 marks): Calculate the call option price in index points.

*c0= 504.799859

Question 2 (8 marks): The below table summarises the borrowing costs confronting two companies.

Borrowing Costs		
	Fixed Rate	Floating Rate
Firm A	4%	6-month LIBOR + 0.6%
Firm B	5%	6-month LIBOR + 0.7%

Suppose Firm A wants to borrow at a floating rate and Firm B wishes to borrow fixed.

Design an intermediated swap that provides a bank with a spread of **30** basis points p.a., and gives the remaining swap benefits **to firm A only**.

Use a clearly labelled diagram to summarise the terms of the arrangement.

*Neither firm has an absolute advantage in the fixed rate market, but firm A is better in the floating rate market. Therefore firm A has a comparative advantage in the floating rate market so it should issue a floating rate bond. Firm B has a comparative advantage in the fixed rate market so it should issue a fixed rate bond.

The total benefit available to all 3 parties including the bank is the absolute value of the difference of differences which is:

TotalBenefitToABAndBank = ||4-5| - |(0.6-0.7)|| = |1 - 0.1| = 0.9%

Subtract the bank's spread top find the benefit to the banks:

TotalBenefitToAandB = 0.9% - 0.3% = 0.6%. Firm A gets all of this benefit.

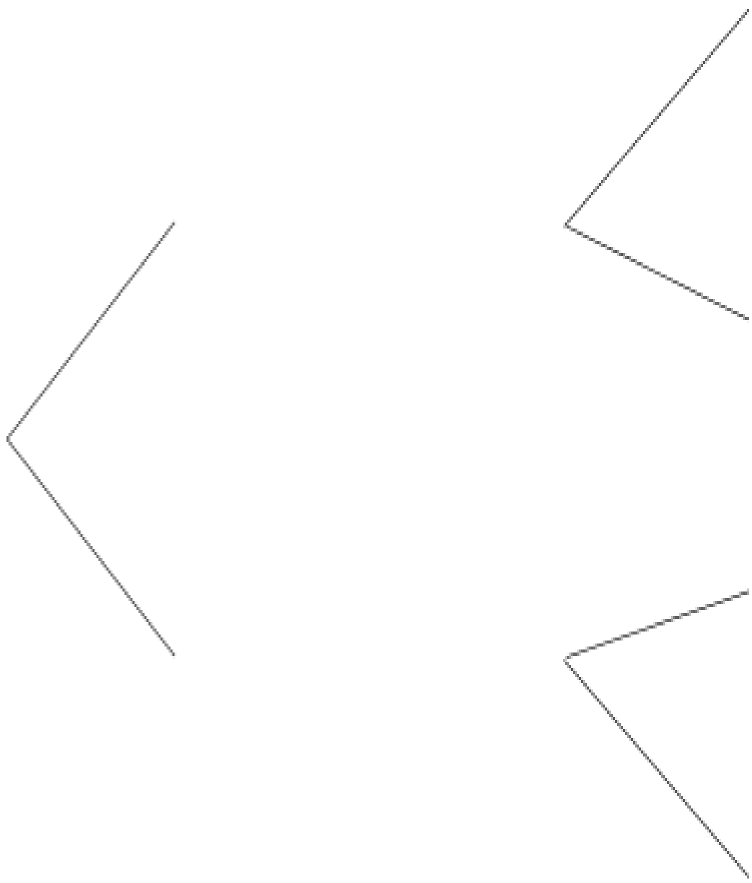
A receives 4-0.6+0.6=4%   B pays 5-0.7+0 =4.3%

A pays 4%   **Firm A**   **Bank**   **Firm B**   B pays L+0.7%

A pays LIB   B receives LIB

Question 4 (total of 8 marks): Consider a 6 month **European** call option with a strike price of $**5**, written on a dividend paying stock currently trading at $**5.50**. The dividend is paid annually and the next dividend is expected to be $**0.30**, paid in **3** months. The risk-free interest rate is **5**% p.a. continuously compounded and the standard deviation of the stock's returns is **40**% p.a..

Calculate the option price now (t=0) using either the no-arbitrage approach or the risk-neutral approach with a two-step binomial tree with 3 months per step. Remember that the option is European so it cannot be exercised before maturity. There are formulas on the formula sheet to help. You may wish to use the binomial tree below to work out the answer.



European

Call (1) or put (-1)	1
T	0.5
sd pa	0.4
t	0.25
Dt, one off paid at t only and not at end	0.3
K	5
r	0.05
u	1.221403
d	0.818731
prob	0.481403
	0.282843
	0.012578

Stock

t=0	t=3mths before div	t=3mths after div	t=6mths	t=6mths option payoff at maturity
			7.838615	2.838615
	6.717715	6.417715		
			5.254381	0.254381
5.5				
			5.133579	0.133579
	4.503019	4.203019		
			3.441141	0

Option

t=0	t=3mths after div	t=6mths option payoff at maturity
		2.838615
	1.479826	
		0.254381
0.736069		
		0.133579
	0.063507	
		0

Question 5 (total of 8 marks): Suppose a stock currently trades at $**100**. The stock's semi-annual dividend is expected to be $**6**, paid in **3** months from now. Assume a **10**% continuously compounded risk-free rate.

Question 5a (3 marks): Calculate the futures price of a **6**-month **futures** contract on this stock, as implied by the above information.

F_0.5 = (S_0*exp(r*3/12) - D_0.25)*exp(r*3/12)

=(100*exp(0.1*3/12) - 6)*exp(0.1*3/12)

=98.97521891

Question 5b (5 marks): If the fair futures price that you calculated above suddenly changed to $**105** but all else was unchanged and there was no news about the company, then explain how you could conduct a risk-free arbitrage. Assume that the future is mis-priced. You're best able to show the steps using an arbitrage table.

Hint: Construct the arbitrage table by having some position in the physical mispriced future above and an offsetting position in a synthetic future. The synthetic future can be constructed using stocks and bonds.

*Short the physical future since it's overpriced. Long the synthetic future (=long stock, and short bond (borrow)) to balance out the risk.

Viewing the below amounts as investments (not cash flows) at time zero, then all positive investments are payments by us now which are buy (long) transactions and all negative investments are receipts to us now which are sell (short) transactions:

$$V_{0\,LF\,synthetic} = S_0 - \frac{K_T}{e^{r.T}}$$

LongSyntheticFuture = LongStock + ShortBond

To find the amounts of these assets that we need to long and short to make a risk free zero capital arbitrage, we'll use an arbitrage table. Note that this arbitrage table shows cash flows, not investments:

Action	**t=0**	**t=3mth**	**t=6mth**
Short physical future	0		-(ST-105)
Long stock	-100	6	ST
Short bond to cover dividend (borrow now)	5.8519 (=6/exp(3/12*0.1)) (Step 4)	-6 (Step 3)	
Short bond to bo(borrow now)	99.87908957 (=105/exp(6/12*0.1)) (Step 5)		-105 (Step 2)
Total	5.730949045 (Step 6)	0 (Step 1)	0 (Step 1)

Formulas

$$\mathrm{r}_{\text{continuously compounded}} = \ln[1 + \mathrm{r}_{\text{discrete}}]$$

$$\mathrm{P}_0 = \frac{\mathrm{P}_\mathrm{t}}{\mathrm{e}^{\mathrm{t.r}_{\text{continuously compounded}}}}$$

$$\mathrm{P}_0 = \frac{\mathrm{P}_\mathrm{t}}{(1 + \mathrm{r}_{\text{discrete}})^\mathrm{t}}$$

$$\mathrm{r}_{\text{discrete}} = \mathrm{e}^{\mathrm{r}_{\text{continuously compounded}}} - 1$$

$$\mathrm{h}^* = \rho_{\mathrm{S,F}}.\frac{\sigma_\mathrm{S}}{\sigma_\mathrm{F}}$$

$$\mathrm{N}^*_{\text{no tailing}} = \mathrm{h}^*.\frac{\mathrm{Q_S}}{\mathrm{Q_F}}$$

$$\mathrm{N}^*_{\text{tailing}} = \mathrm{h}^*.\frac{\mathrm{V_S}}{\mathrm{V_F}}$$

$$F_{0,T} = S_0.e^{r.T}$$

$$f_{0,long} = S_0 - K_T.e^{-r.T}$$

$$f_{long} = -f_{short}$$

$$p = \frac{e^{rt} - d}{u - d}$$

$$u = e^{\sigma\sqrt{t}}$$

$$d = \frac{1}{u} = e^{-\sigma\sqrt{t}}$$

$$c_0 + K.e^{-r.T} = p_0 + S_0$$

$$c_0 = S_0.N[d_1] - K.e^{-r.T}.N[d_2]$$

$$p_0 = -S_0.N[-d_1] + K.e^{-r.T}.N[-d_2]$$

$$\mathrm{d}_1 = \frac{\ln[\mathrm{S}_0/K] + (r + \sigma^2/2).T}{\sigma.\mathrm{T}^{0.5}}$$

$$\mathrm{d}_2 = \mathrm{d}_1 - \sigma.\mathrm{T}^{0.5} = \frac{\ln[\mathrm{S}_0/K] + (r - \sigma^2/2).T}{\sigma.\mathrm{T}^{0.5}}$$

$$\Delta_{\text{call}} = \frac{\partial c}{\partial S} = N[d_1]$$

$$\Gamma_{\text{call}} = \Gamma_{\text{put}} = \frac{\partial \Delta_{\text{call}}}{\partial S} = \frac{\partial^2 c}{\partial S^2} = \frac{\left(\frac{1}{(2.\pi)^{1/2}}.e^{-x^2/2}\right)}{S_0.\sigma.T^{1/2}}$$

$$\Theta_{\text{call}} = \frac{\partial c}{\partial T} = \frac{\left(S_0.\frac{1}{(2.\pi)^{1/2}}.e^{-x^2/2}.\sigma\right)}{2.T^{1/2}} - r.K.e^{r.T}.N[d_2]$$

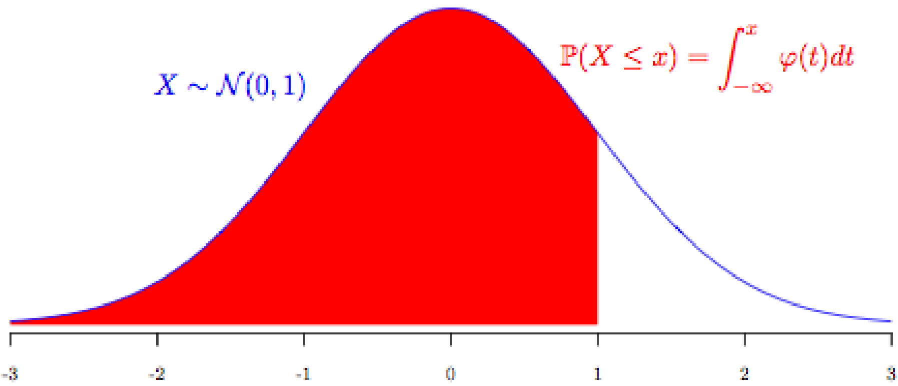
$$\Theta_{\text{put}} = \frac{\partial p}{\partial T} = \frac{\left(S_0.\frac{1}{(2.\pi)^{1/2}}.e^{-x^2/2}.\sigma\right)}{2.T^{1/2}} + r.K.e^{r.T}.N[-d_2]$$

$$c_{t+h} \approx c_t + \epsilon.\Delta_{\text{call}}[S_t] + \frac{1}{2}.\epsilon^2\Gamma_{\text{call}}[S_t] + h.\Theta_{\text{call}}[S_t]$$

$$VaR_{prob} = -V.(\mu + \alpha_{prob}.\sigma)$$

$$\alpha = \phi^{-1}[1 - prob] = NormsInv[1 - prob]$$

$$ES[prob] = \mu + \sigma.\frac{\phi\left[\phi^{-1}[1 - prob]\right]}{1 - prob}$$



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990