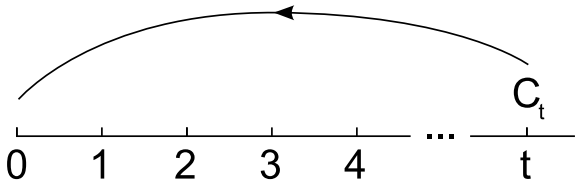
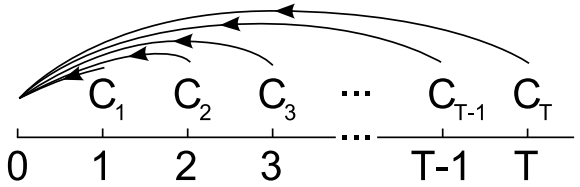
Present Value Formulas

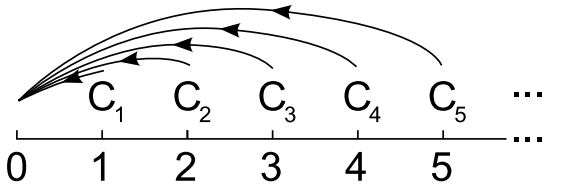
$$PV[single\ cash\ flow] = V_0 = \frac{C_t}{(1 + r_{eff})^t}$$



$$PV[annuity] = V_0 = \frac{C_1}{r_{eff}}\left(1 - \frac{1}{(1+r_{eff})^T}\right)$$



$$PV[perpetuity] = V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$



Present Value of a Single Cash Flow

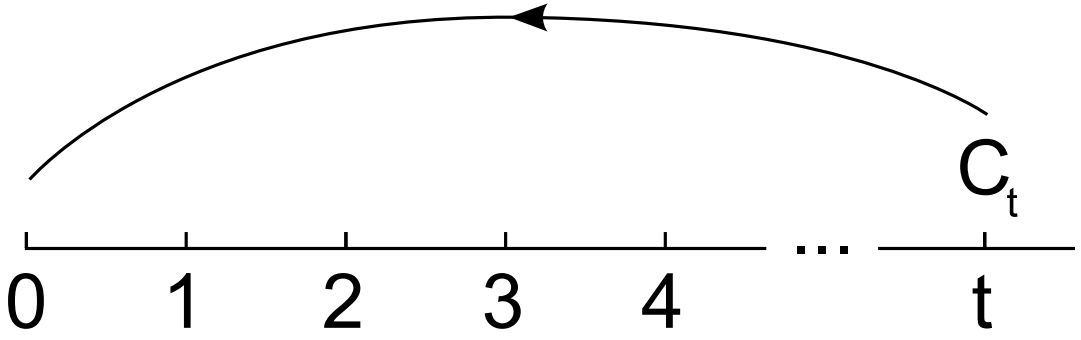
$$\boldsymbol{PV[\ single\ cash\ flow\ ] = V_0 = \frac{C_t}{(1+r)^t}}$$

Where:

C_t = cash flow at time t.

t = time periods.

r = the effective rate over a single period.



Calculation Example: Present Value of a Single Cash Flow

Question: What is the present value of $100 received in 5 years when interest rates are 8% pa?

Answer:

$$V_0 = \frac{C_t}{(1+r)^t}$$

$$= \frac{100}{(1+0.08)^5}$$

$$= 68.0583$$

***Future* Value of a Single Cash Flow**

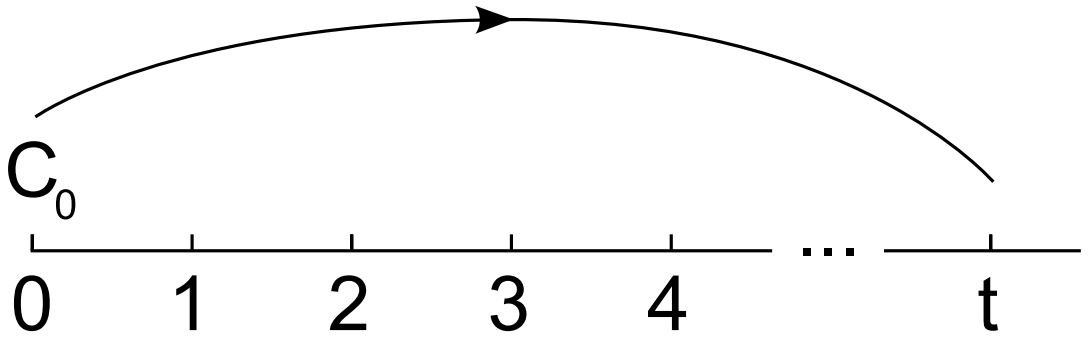
$$\boldsymbol{FV[\ single\ cash\ flow\ ] = V_t = C_0(1+r)^t}$$

Where:

$C_0 =$ cash flow now.

$t =$ time periods into the future.

$r =$ the effective rate over a single period.



Calculation Example: Future Value of a Single Cash Flow

Question: You have \$100 in the bank. Interest rates are 8% pa. How much will you have in the bank after 5 years?

Answer:

$$V_t = C_0(1+r)^t$$

$$= 100 \times (1+0.08)^5$$

$$= 146.9328$$

Calculation Example: Present and Future Values

Question: If you pay this year's university fees of $5,000 now, the government will give you a 25% discount. Otherwise the government will lend you the $5,000, but will capitalise interest charges (add interest charges to the principal) at the rate of inflation which is expected to be 2.5% pa, compounding annually. When you start work, the government will demand repayment of your debt. You expect to start work and have to pay off all of your debt in a single payment in 6 years.

You can borrow and lend at 8% pa from and to the bank.

Should you pay your fees now or in 6 years?

Answer:

Option 1: Pay your uni fees now.

If you pay your fees now and receive the 25% discount, they will cost:

$$V_0 = 5{,}000 \times (1 - 0.25)$$

$$= \mathbf{3{,}750}$$

Option 2: Pay your university fees in 6 years.

The future value of the uni fees in 6 years, growing at the inflation rate of 2.5% pa will be:

$$V_t = C_0(1 + r)^t$$

$$V_6 = 5{,}000 \times (1 + 0.025)^6 = \mathbf{5{,}798.4671}$$

This is the amount that the government will demand for repayment. A naive person would compare this to the $3,750 and conclude that paying immediately is better, but that is wrong. The two amounts can't be compared since they are at different times. Values can only be compared at the same point in time.

The present value of $5,798.47 can be calculated and compared to the cost of paying the fees now.

$$V_0(1+r)^t = V_6$$

$$V_0(1+0.08)^6 = 5{,}798.4671$$

$$V_0 = \frac{5{,}798.4671}{(1+0.08)^6} = \mathbf{3{,}654.0178}$$

Therefore you should pay your fees in 6 years since it has a lower present value of costs.