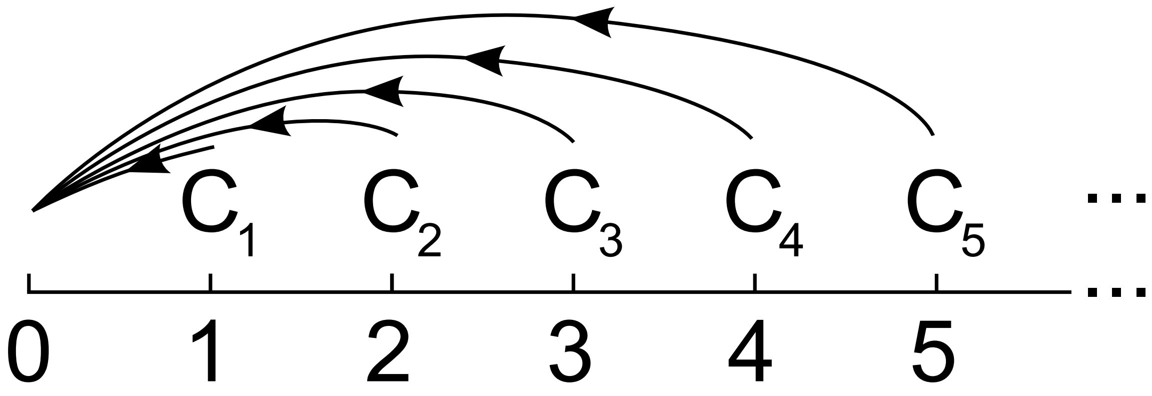
Present Value of a Perpetuity

The 'perpetuity with growth' formula finds the present value of periodic cash flows that continue forever.

This formula is also called the 'Gordon Growth Model' or the 'Dividend Discount Model' (DDM).

$$\boldsymbol{PV[perpetuity\ with\ growth] = P_0 = \frac{C_1}{r-g}}$$

C_1 = dividend cash flow received at $t = 1$. The dividend cash flows go on forever, but grow by g every period.



g = effective growth rate of the dividend over a single period.

r = effective discount rate over a single period, also called the required return on equity.

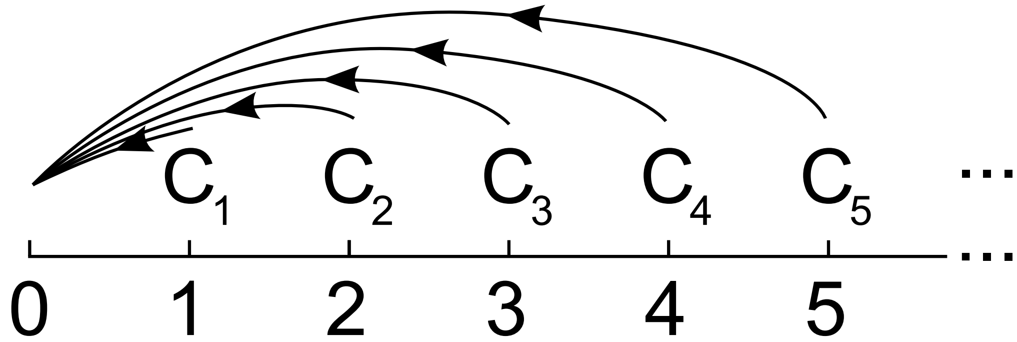
Note that C_1 is used instead of C to remind you that the first cash flow is 1 period ahead of the present value V_0. The perpetuity formula does not include a cash flow at $t = 0$.

The cash flows grow by g forever:

$C_2 = C_1(1 + g)$

$C_3 = C_1(1 + g)^2$

$C_4 = C_1(1 + g)^3$ and so on.



Perpetuities with no growth are called level perpetuities. In this case, $g = 0$ and $C_1 = C_2 = C_3$ and so on.

Calculation Example: Present Value of a Perpetuity (with no growth)

Question: Your friend promises to pay you \$50 at the end of every year forever, if you lend him \$400 now. Today is new year's day so the next payment will be in one year. Interest rates are 10% pa. Is this a good deal for you?

Answer: Your friend is offering you a perpetuity with no growth.

$$V_0 = \frac{C_1}{r - g}$$

$$= \frac{50}{0.1 - 0} = 500$$

$500 is more than $400 so it's a good deal.

But this is assuming that your friend actually does pay you forever even after he becomes old and senile. Assuming he stops paying you in 40 years, then the $50 will be an annuity:

$$V_0 = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$= \frac{50}{0.1}\left(1 - \frac{1}{(1+0.1)^{40}}\right)$$

$$= 488.9525$$

This is more than $400, so it's still a good deal.

Calculation Example: Perpetuity

Question 1: A start-up company is forecast to pay its first dividend of \$1 per share in 5 years. From then on, this annual dividend will grow by 2% pa. The required return on the stock is 10% pa. All rates are given as effective annual rates. What is the value of the stock?

Answer: This question is slightly trickier than simply applying the DDM since the first cash flow is not 1 year away, it is 5 years away.

What we will do is value the stock using the DDM which will give a value at t=4, one year before the first dividend at t=5, then discount that price back to the present (t=0).

$$V_0 = \frac{C_1}{r_{eff} - g_{eff}}$$

$$V_4 = \frac{C_5}{r_{eff} - g_{eff}}$$

$$= \frac{1}{0.1 - 0.02} = 12.50$$

But this is a value at t=4. To find the current price at t=0,

$$V_0 = \frac{V_4}{\left(1 + r_{eff}\right)^4}$$

$$= \frac{12.50}{(1 + 0.1)^4} = 8.537668192$$

So the stock price should be $8.54 right now