NPV and Projects with Different Lives

How do we choose between mutually exclusive projects when one ends before the other? For example, there are two machines that do the same job and the expensive machine lasts for longer than the cheap one. Which do you choose?

Wrong method 1: Choose the machine with the lowest present value of costs. Wrong because obviously the cheap machine will always appear better even though it lasts for less time.

Wrong method 2: Divide each machine's present value of costs by the number of years it lasts for. Wrong because it ignores the time value of money, it is an accounting way of thinking. It is spreading the present value of costs, which is an amount now, across years in the future which should actually be grown to account for the time value of money.

Equivalent Annual Cost (EAC)

Correct method: Find the present value of each machine's costs. Then calculate the Equivalent Annual Cost (EAC) using the annuity formula which spreads the costs into equal payments over the life of each machine, taking timing into account. Choose the machine with the lowest EAC.

This method implicitly assumes that whichever machine the firm chooses, it will keep buying identical machines to replace the previous one forever. This is known as 'constant chain of replacement'.

$$PV[annuity] = V_0 = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$V_0 = C_1 \times \frac{1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$V_0 = \boldsymbol{C_1} \times AnnuityFactor[r, T]$$

So V_0 will be the NPV of all future costs, C_1 will be the equivalent annual cost, and the annuity factor which depends on the effective discount rate per period *r* and the number of periods T.

$$\mathrm{NPV[all\ costs]} = \mathbf{EAC} \times \mathrm{AnnuityFactor[r, T]}$$

$$\mathrm{EAC} = \frac{\mathrm{NPV[all\ costs]}}{\mathrm{AnnuityFactor[r, T]}} = \frac{\mathrm{V_0}}{\frac{1}{\mathrm{r}}\left(1 - \frac{1}{(1+\mathrm{r})^{\mathrm{T}}}\right)}$$

Calculation Example: EAC

Question 7: You estimate the following cash flows from buying, maintaining and selling two different old cars. Which one is the better choice if the discount rate is 10%? Ignore taxes and assume 'constant chain of replacement'.

	Cash Flows ($)	
Time	Toyota Camry	Holden Commodore
0	-1800	-3500
1	-500	-400
2	-500	-400
3	-500	-400
4	-500	-400
5	+50	-400
6		-400
7		+1500

Answer:

First find the total NPV of each car's costs:

$$V_{0,toyota} = -1800 - \frac{500}{0.1}\left(1 - \frac{1}{(1+0.1)^4}\right) + \frac{50}{(1+0.1)^5}$$
$$= -3{,}353.90$$

$$V_{0,holden} = -3500 - \frac{400}{0.1}\left(1 - \frac{1}{(1+0.1)^6}\right) + \frac{1500}{(1+0.1)^7}$$
$$= -4{,}472.40$$

Now find the equivalent annual costs.

For the Toyota:

$$-3{,}353.90 = \frac{\mathrm{C}_{1,\mathrm{toyota}}}{0.1}\left(1 - \frac{1}{(1+0.1)^5}\right)$$

$$\mathrm{C}_{1,\mathrm{toyota}} = -884.75$$

So 884.75 is the Toyota's equivalent annual cost.

For the Holden:

$$-4{,}472.40 \ = \frac{\mathrm{C}_{1,\mathrm{holden}}}{0.1}\left(1 - \frac{1}{(1+0.1)^7}\right)$$

$$\mathrm{C}_{1,\mathrm{holden}} = -918.65$$

So 918.65 is the Holden's equivalent annual cost.

Choose the Toyota since it has a lower equivalent annual cost.

Questions: Equivalent Annual Cash Flow

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