Debt

- Lending and borrowing as buying and selling debt.
- APR's versus effective rates.
- Home loans:
    - Fully amortising loans.
    - Interest only loans.
- Bond pricing exactly one period before the next coupon.
- Bond pricing between coupons.
- Short term wholesale (money market) debt and overview.
- Yield curves and the term structure on interest rates.

Borrowing: Synonyms

Borrowers receive money at the start. Borrowing is equivalent to selling a debt contract.

To remember that a borrower sells debt, just think about how a borrower receives cash at the **start**. If you're in a shop and you receive cash, that's because you're the seller, not the buying customer.

If a lady sells debt to a man, she receives cash now in return for selling her promise to pay him back the principal and interest later. She sells the debt contract, she is the borrower.

Borrowing is also known as:

- Selling debt (receiving money at the start)
- Having a debt liability
- Issuing debt (issuing the debt contract at the start)
- Being short debt (shorting is a synonym of selling)
- Owing money (a borrower ‘owes’ money)
- Being in debt
- Being a debtor. Or less commonly, a creditee.
- Writing debt (since you write the contractual promise to pay back principal and interest and give the paper contract to the lender)

Lending: Synonyms

Lenders give money at the start. Lending is equivalent to buying a debt contract.

To remember that a lender buys debt, just think about how a lender gives cash at the **start**. If you're in a shop and you pay cash, that's because you're the buying customer, not the selling shop keeper.

If a man buys debt from a lady, he pays cash now to buy her promise to pay him back the principal and interest later. He buys the debt contract, he is the lender.

Lending is also known as:

- Buying debt (giving money at the start)
- Having a debt asset or owning a debt asset
- Investing in debt
- Being long debt (longing is a synonym of buying, shorting is selling)
- Being a creditor. Or less commonly, a debtee.
- Being owed money. Note that this is **not** the past tense of owe. "He is owed money" means the man is a lender. "She owes money" means the lady is a borrower.

Questions: Debt Terminology

http://www.fightfinance.com/?q=128,129,130,374,234,372,373,

Annualised Percentage Rates (APR's)

Most interest rates are quoted as Annualised Percentage Rates (APR's). This is both by convention and in some countries by law. This is true for credit card rates, mortgage rates, bond yields, and many others.

The compounding period of an APR is not always explicitly stated. However, it can usually be assumed that **the compounding frequency of an APR is the same as the payment frequency**.

For example, a credit card might advertise an interest rate of 24% pa. This must be an APR since all advertised rates have to be APR's by law. Because credit cards are always paid off monthly, the compounding frequency is per month. Therefore the interest rate is 24% pa given as an APR compounding monthly.

While APR’s are the rate that you see quoted and advertised, unfortunately they **cannot** be used to find present or future values of cash flows. You must convert the APR to an effective rate before doing financial mathematics.

Confusion: APR's are sometimes called nominal rates. Unfortunately, nominal has another meaning related to inflation (nominal versus real returns). We will avoid calling APR's nominal rates in these notes.

Effective Rates

Effective rates compound only once over their time period, and the time period can be of any length, not necessarily annual.

Effective rates can be used to discount cash flows.

APR's **cannot** be used to discount cash flows, they must be converted to effective rates first.

Note that all of the calculation examples up to here have assumed that the interest rate given is an effective rate.

Calculation Example: Present Values and Effective Rates

Question: What is the present value of receiving \$100 in one year when the effective monthly rate is 1%?

Answer: Since the effective interest rate is per month, the time period must also be in months, so:

$$V_o = \frac{C_t}{(1+r)^t}$$

$$= \frac{100}{(1+0.01)^{12}}$$

$$= 88.7449$$

APR's and Effective Rates

- An APR compounding monthly is equal to 12 multiplied by the effective monthly rate.

  $r_{APR\ comp\ monthly} = r_{eff\ monthly} \times 12$

- An APR compounding semi-annually is equal to 2 multiplied by the effective 6 month rate.

  $r_{APR\ comp\ per\ 6mths} = r_{eff\ 6mth} \times 2$

- An APR compounding daily is equal to 365 multiplied by the effective daily rate.

  $r_{APR\ comp\ daily} = r_{eff\ daily} \times 365$

Example: Future Values with APR's

Question: How much will your credit card debt be in 1 year if it's $1,000 now and the interest rate is 24% pa?

Wrong Answer: $V_t = C_0(1+r)^t = 1000(1+0.24)^1 = 1{,}240$

Answer: Since credit cards are paid off per month and rates are by default given as APR’s, the 24% must be an APR compounding monthly. Therefore the effective monthly rate will be the APR divided by 12.

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12} = \frac{0.24}{12} = 0.02$$

$$V_{12mths} = C_0\left(1 + r_{eff\ monthly}\right)^{t_{months}}$$

$$= 1000(1+0.02)^{12} = 1{,}268.2418$$

Converting Effective Rates To Different Time Periods

Compounding the rate higher (up to a longer time period):

$$r_{eff\ annual} = \left(1 + r_{eff\ monthly}\right)^{12} - 1$$

$$r_{eff\ semi\text{-}annual} = \left(1 + r_{eff\ monthly}\right)^{6} - 1$$

$$r_{eff\ quarterly} = \left(1 + r_{eff\ monthly}\right)^{3} - 1$$

Compounding the rate lower (down to a shorter time period):

$$r_{eff\ monthly} = \left(1 + r_{eff\ annual}\right)^{\frac{1}{12}} - 1$$

$$r_{eff\ daily} = \left(1 + r_{eff\ annual}\right)^{\frac{1}{365}} - 1$$

Calculation Example: Converting Effective Rates

Question: A stock was bought for $10 and sold for $15 after 7 months. No dividends were paid. What was the effective annual rate of return?

Answer:

First we find the return over 7 months. This will be the effective 7 month rate of return. Note that the time period is in 7-month blocks:

$$V_0 = \frac{V_t}{(1+r)^t}$$

$$V_0 = \frac{V_{7months}}{\left(1 + r_{eff\ 7month}\right)^{1_{seven\ month\ period}}}$$

$$10 = \frac{15}{\left(1 + r_{eff\ 7month}\right)^1}$$

$$\left(1 + r_{eff\ 7month}\right)^1 = \frac{15}{10}$$

$r_{eff\ 7month} = \frac{15}{10} - 1 = 0.5 = 50\%$, which is the effective 7 month rate.

Now we need to convert the effective 7 month rate to an effective annual rate (EAR). This can be done by ‘compounding up’ by 12/7 in one step:

$$r_{eff\ annual} = \left(1 + r_{eff\ 7mth}\right)^{\frac{12}{7}} - 1$$

$$= (1 + 0.5)^{12/7} - 1 = 1.0039 = 100.39\%$$

Or it can be broken down into steps:

- Compounding the 7-month rate down to a monthly rate:

$$r_{eff\ monthly} = \left(1 + r_{eff\ 7mth}\right)^{1/7} - 1$$

$$= (1 + 0.5)^{1/7} - 1 = 0.059634 = 5.9634\%$$

- Then compound the monthly rate up to a 12-month (annual) rate:

$$r_{eff\ annual} = \left(1 + r_{eff,monthly}\right)^{12} - 1$$
$$= (1 + 0.059634)^{12} - 1 = 1.0039 = 100.39\%$$

Calculation Example: Converting APR's to Effective Rates

Question: You owe a lot of money on your credit card. Your credit card charges you 9.8% pa, given as an APR compounding per month.

You have the cash to pay it off, but your friend wants to borrow money from you and offers to pay you an interest rate of 10% pa given as an effective annual rate. Assume that your friend will definitely pay you back (no credit risk).

Should you use your cash to pay off your credit card or lend it to your friend?

Answer: The loan's 10% effective annual rate can't be immediately compared to the credit card's 9.8% APR compounding per month.

Method 1: Convert the credit card's 9.8% APR compounding per month to an effective annual rate:

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12} = \frac{0.098}{12} = 0.0081667$$

$$\begin{aligned} r_{eff\ annual} &= \left(1 + r_{eff\ monthly}\right)^{12} - 1 \\ &= (1 + 0.0081667)^{12} - 1 = 0.1025 = 10.25\% \end{aligned}$$

So the credit card's 9.8% APR compounding per month converts to an effective annual rate of 10.25%. This is more than the loan's 10% effective annual rate.

You should pay off your credit card which costs 10.25% rather than lend to your friend which only earns 10%, where both rates are effective annual rates.

Method 2: Convert the loan's 10% effective annual rate to an APR compounding per month:

$$r_{eff\ monthly} = \left(1 + r_{eff\ annual}\right)^{1/12} - 1$$

$$= (1 + 0.1)^{1/12} - 1 = 0.00797414043$$

$$r_{APR\ comp\ monthly} = r_{eff\ monthly} \times 12$$

$$= 0.00797414043 \times 12$$

$$= 0.09568968514 = 9.568968514\%\ pa$$

So the 10% effective annual rate that you can lend at converts to a 9.569% APR compounding per month. This is less than your 9.8% pa cost of funds using your credit card, where both are APR's compounding monthly, so don't lend to your friend.

Calculation Examples

Question: Assume 30 days per month and 360 days per year.

Convert a **9.8**% APR compounding per **month** into the following:

$$r_{eff\ 6month} = \left(1 + \frac{0.098}{12}\right)^{6} - 1 = 0.050011377 \text{ per 6 months}$$

$$r_{APR\ comp\ 6months} = \left(\left(1 + \frac{0.098}{12}\right)^{6} - 1\right) \times 2 = 0.100022754 \text{ pa}$$

$$r_{eff\ daily} = \left(1 + \frac{0.098}{12}\right)^{\frac{1}{30}} - 1 = 0.000271153 \text{ per day}$$

$$r_{APR\ comp\ daily} = \left(\left(1 + \frac{0.098}{12}\right)^{\frac{1}{30}} - 1\right) \times 360 = 0.097615231 \text{ pa}$$

Calculation Examples

Question: Assume 30 days per month and 360 days per year.
Convert a **10**% effective annual rate ($r_{eff\ annual}$) into the following:

$$r_{eff\ 6month} = (1+0.1)^{6/12} - 1 = 0.048808848 \text{ per 6 months}$$

$$r_{APR\ comp\ 6months} = \left((1+0.1)^{6/12} - 1\right) \times 2 = 0.097617696 \text{ pa}$$

$$r_{eff\ daily} = (1+0.1)^{1/360} - 1 = 0.000264786 \text{ per day}$$

$$r_{APR\ comp\ daily} = \left((1+0.1)^{1/360} - 1\right) \times 360 = 0.095322798 \text{ pa}$$

$$r_{eff\ 2\ year} = (1+0.1)^{2} - 1 = 0.21 \text{ per 2 years}$$

$$r_{eff\ 1\ second} = (1+0.1)^{1/(12\times30\times24\times60\times60)} - 1 = 0.000000003064/s$$

$$r_{APR\ comp\ second} = r_{eff\ 1\ second} \times 12 \times 30 \times 24 \times 60 \times 60$$
$$= 0.0953101823\ pa \approx ln(1+0.1) = r_{continuously\ compounded\ pa}$$

Questions: APR's and Effective Rates

http://www.fightfinance.com/?q=290,330,16,26,131,49,64,265,

Fully Amortising Loans

Most home loans are fully amortising or 'Principal and Interest' (P&I) loans.

When a fully amortising home loan borrower makes their last payment, the loan is fully paid off.

We can value fully amortising home loans using the annuity formula assuming that the interest rate r is expected to be constant.

$$V_{0\ fully\ amortising} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

Example: Fully Amortising Loans

Question: Mortgage rates are currently 6% and are not expected to change.

You can afford to pay $2,000 a month on a mortgage loan.

The mortgage term is 30 years (matures in 30 years).

What is the most that you can borrow using a fully amortising mortgage loan?

Answer: Since the mortgage is fully amortising, at the end of the loan's maturity the whole loan will be paid off.

The bank will lend you the present value of your monthly payments for the next 30 years. This can be calculated using the annuity formula.

The $2,000 payments are monthly, therefore the interest rate and time periods need to be measured in months too.

$$t = 30 \times 12 = 360\, months$$

$$r_{eff\ monthly} = \frac{r_{APR\ comp\ monthly}}{12} = \frac{0.06}{12} = 0.005 = 0.5\%$$

$$V_{0\ fully\ amortising} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$= \frac{2000}{0.005}\left(1 - \frac{1}{(1+0.005)^{360}}\right) = \$333{,}583$$

Example: Fully Amortising Loans

Question: You wish to borrow $**10,000** for **2** years as an unsecured personal loan.

Interest rates are quite expensive at **60**% pa and are not expected to change.

What will be your monthly payments on a fully amortising loan?

Answer:

$$V_{0\ fully\ amortising} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

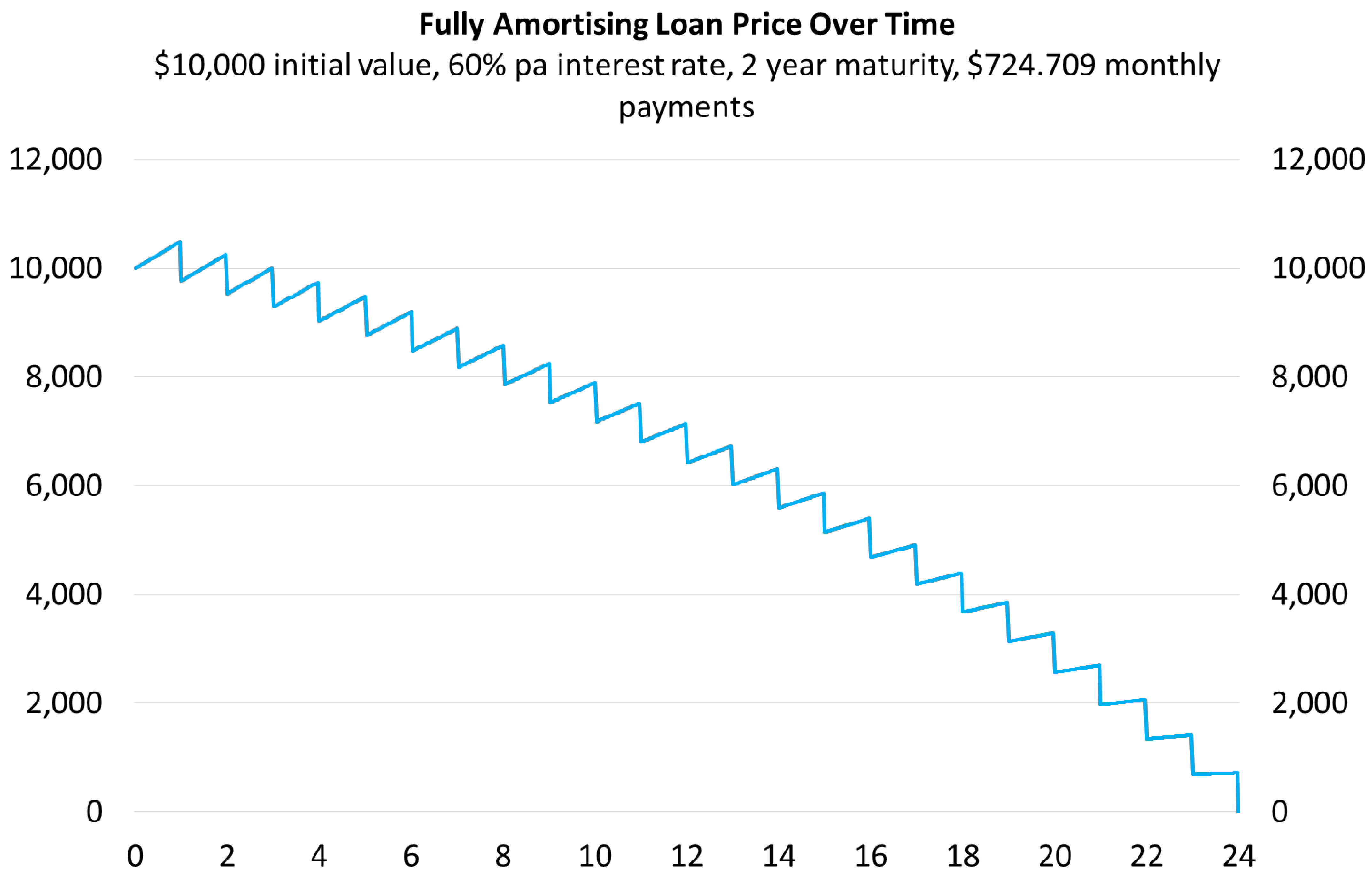
$$10000 = \frac{C_1}{0.6/12}\left(1 - \frac{1}{(1+0.6/12)^{2\times 12}}\right)$$

$$10000 = C_1 \times \frac{1}{0.6/12}\left(1 - \frac{1}{(1+0.6/12)^{2\times 12}}\right)$$

$$C_1 = \frac{10000}{\frac{1}{0.6/12}\left(1 - \frac{1}{(1+0.6/12)^{2\times 12}}\right)}$$

$$= \frac{10000}{13.7986417943}$$

$$= 724.709007529$$


Fully Amortising Loan Price Over Time
$10,000 initial value, 60% pa interest rate, 2 year maturity, $724.709 monthly payments
12,000
10,000
8,000
6,000
4,000
2,000
0
0
2
4
6
8
10
12
14
16
18
20
22
24

Interest and Principal Components

Each total loan payment (C_{total}) can be broken into interest ($C_{interest}$) and principal ($C_{principal}$) components:

$$C_{total} = C_{interest} + C_{principal}$$

The interest component at time one ($C_{1interest}$) is defined as the interest rate over the first period ($r_{0\rightarrow 1}$) multiplied by the initial (time zero) loan price or value (V_0):

$$C_{1interest} = V_0 . r_{0\rightarrow 1}$$

The principal component is the remaining part:

$$C_{1principal} = C_{total} - C_{1interest}$$

For a general formula, replace time 1 with t and 0 with $t - 1$.

Calculation Example: Loan Schedule

Fully Amortising Home Loan Schedule				
$1 million initial value, 3.6% pa interest rate, 30 year maturity, monthly payments				
Time months	Value $	Total payment $/month	Interest component $/month	Principal component $/month
0	1000000.00			
1	998453.55	4546.45	3000.00	1546.45
2	996902.45	4546.45	2995.36	1551.09
3	995346.71	4546.45	2990.71	1555.75
4	993786.29	4546.45	2986.04	1560.41
5	992221.20	4546.45	2981.36	1565.09
…	…	…	…	…
357	13557.93	4546.45	54.15	4492.30
358	9052.15	4546.45	40.67	4505.78
359	4532.85	4546.45	27.16	4519.30
360	0.00	4546.45	13.60	4532.85

$$C_{1total} = \frac{V_0}{\frac{1}{r_{eff\ monthly}}\left(1-\frac{1}{(1+r_{eff\ monthly})^{T_{months}}}\right)}$$

$$= \frac{1{,}000{,}000}{\frac{1}{\left(\frac{0.036}{12}\right)}\left(1-\frac{1}{\left(1+\frac{0.036}{12}\right)^{30\times12}}\right)} = 4{,}546.45$$

$$C_{1interest} = V_0 . r_{eff\ monthly,0\to1} = V_0 . \frac{r_{APR\ comp\ monthly,0\to1}}{12}$$

$$= 1{,}000{,}000 \times \frac{0.036}{12} = 3{,}000$$

$$C_{1principal} = C_{1total} - C_{1interest}$$

$$= 4{,}546.45 - 3{,}000$$

$$= 1{,}546.45$$

Fully Amortising Home Loan Schedule

3.6% pa interest rate, 30 year maturity, monthly payments

Time months	Value $	Total payment $/month	Interest component $/month	Principal component $/month
0	1000000.00			
1	998453.55	4546.45	3000.00	1546.45
2	996902.45	4546.45	2995.36	1551.09
3	995346.71	4546.45	2990.71	1555.75
4	993786.29	4546.45	2986.04	1560.41
5	992221.20	4546.45	2981.36	1565.09
...	...	...	...	...
357	13557.93	4546.45	54.15	4492.30
358	9052.15	4546.45	40.67	4505.78
359	4532.85	4546.45	27.16	4519.30
360	0.00	4546.45	13.60	4532.85

Loan Valuation: Prospective vs Retrospective

Fully Amortising Home Loan Schedule				
3.6% pa interest rate, 30 year maturity, monthly payments				
Time	Value	Total payment	Interest component	Principal component
months	$	$/month	$/month	$/month
0	1000000.00			
1	998453.55	4546.45	3000.00	1546.45
2	996902.45	4546.45	2995.36	1551.09
3	995346.71	4546.45	2990.71	1555.75
4	993786.29	4546.45	2986.04	1560.41
5	992221.20	4546.45	2981.36	1565.09
...	...	...	...	...
357	13557.93	4546.45	54.15	4492.30
358	9052.15	4546.45	40.67	4505.78
359	4532.85	4546.45	27.16	4519.30
360	0.00	4546.45	13.60	4532.85

Prospective loan valuation: The value (price) of any asset is the present value of its future cash flows.

So discount the future cash flows to the present. For example, to find the month 1 loan value just after that first payment:

$$V_1 = \frac{C_2}{r_{eff\,monthly}}\left(1 - \frac{1}{(1+r_{eff\,monthly})^{T_{months\,remaining}}}\right)$$

$$= \frac{4,546.4535}{\left(\frac{0.036}{12}\right)}\left(1 - \frac{1}{\left(1+\frac{0.036}{12}\right)^{30\times12-1}}\right) = 998,453.55$$

Retrospective loan valuation: Deduct the principal portion of the loan payment from the prior value:

$$V_1 = V_0 - C_{1principal}$$

$$= 1{,}000{,}000 - 1{,}546.45 = 998{,}453.55$$

Fully Amortising Home Loan Schedule

3.6% pa interest rate, 30 year maturity, monthly payments

Time months	Value $	Total payment $/month	Interest component $/month	Principal component $/month
0	1000000.00			
1	998453.55	4546.45	3000.00	1546.45
2	996902.45	4546.45	2995.36	1551.09
3	995346.71	4546.45	2990.71	1555.75
4	993786.29	4546.45	2986.04	1560.41
5	992221.20	4546.45	2981.36	1565.09
…	…	…	…	…
357	13557.93	4546.45	54.15	4492.30
358	9052.15	4546.45	40.67	4505.78
359	4532.85	4546.45	27.16	4519.30
360	0.00	4546.45	13.60	4532.85

Comment: Prospective over Retrospective

The retrospective method only works when credit risk remains unchanged and everything goes to plan.

Using the home loan in the previous example, say a volcano explodes underneath the borrower's house and they lose their job at month 4 just after the payment at that time, and they only have enough savings for one more monthly loan payment.

The **prospective** method would correctly value the month 4 loan as the present value of that one remaining $4,546.45 month 5 payment, since the remaining payments at month 6, 7 and so on will not occur and there's no property to re-possess.

But the **retrospective** method would incorrectly value the month 4 loan at $993,786.29 which is far too high.

The retrospective method should be discouraged since past cash flows are sunk and shouldn't be considered when pricing assets. Always think about the future, not the past.

Questions: Fully Amortising Loans

http://www.fightfinance.com/?q=19,87,134,149,172,187,203,204,222,259,

Interest-Only Loans

Borrowers with interest-only loans don't make any principal payments. Their entire loan payments are interest. So the loan is not paid off until the very last payment when the original amount must be fully repaid, plus the interest over that last month.

Often this big amount is paid off by re-financing (rolling over) the debt which means borrowing again, or by selling the house.

Most interest-only loans in Australia are actually interest-only for 5 years or less. After this, they convert to fully amortising loans.

Interest-Only Loans Valuation

Interest only loans are easily valued using the 'perpetuity without growth' formula:

$$V_{0\ interest\ only} = \frac{C_1}{r} = \frac{C_{1\ monthly}}{r_{eff\ monthly}} = \frac{C_{1\ monthly}}{\left(\frac{r_{APR\ comp\ monthly}}{12}\right)}$$

This makes sense since an interest-only loan that is repeatedly refinanced always has the same principal, it's never paid off. If the interest rate is constant forever, then the borrower will pay constant interest cash flows ($C_1 = C_2 = C_3 = \cdots$) perpetually.

Another way of looking at the interest-only loan is to assume that the final principal payment is paid off without refinancing. The initial price (V_0) is the present value of the interest payments (C_1, C_2,…) and *also* the final principal (V_T):

$$\boldsymbol{V_0} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{V_T}{(1+r)^T}$$

Substitute $V_T = \boldsymbol{V_0}$

$$\boldsymbol{V_0} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right) + \frac{\boldsymbol{V_0}}{(1+r)^T}$$

$$\boldsymbol{V_0} - \frac{\boldsymbol{V_0}}{(1+r)^T} = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$\boldsymbol{V_0}\left(1 - \frac{1}{(1+r)^T}\right) = \frac{C_1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

$$\boldsymbol{V_0} = \frac{C_1}{r}$$

Calculation Example: Interest Only Loans

Question: Using the same data as before, what is the most that you can borrow using an interest only mortgage loan? Again, assume constant interest rates.

Answer: Interest-only loans are equivalent to perpetuities with no growth.

$$V_{0\ interest\ only} = \frac{C_1}{r} = \frac{C_{1\ monthly}}{r_{eff\ monthly}} = \frac{C_{1\ monthly}}{\left(\frac{r_{APR\ comp\ monthly}}{12}\right)}$$

$$= \frac{2000}{(0.06/12)} = \$400{,}000$$

Calculation Example: Interest Only Loans

Question: You wish to borrow $**10,000** for **2** years as an unsecured personal loan.

Interest rates are quite expensive at **60**% pa and are not expected to change.

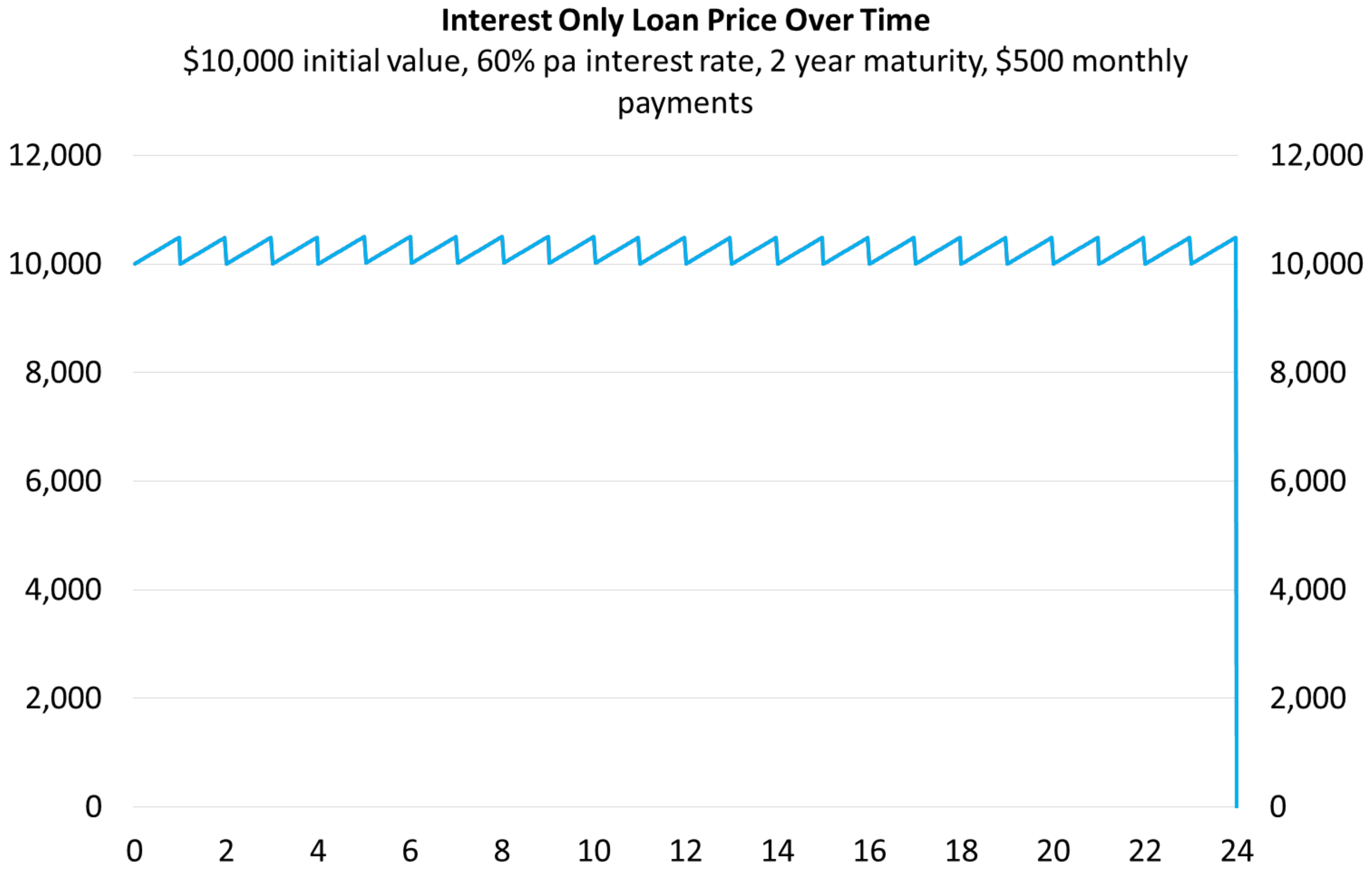
What will be your monthly payments on an interest-only loan?

Answer:

$$V_{0\ finterest\ only} = \frac{C_1}{r}$$

$$10000 = \frac{C_1}{0.6/12}$$

$$C_1 = 10000 \times 0.6/12 = 500$$


Interest Only Loan Price Over Time
$10,000 initial value, 60% pa interest rate, 2 year maturity, $500 monthly payments
12,000
10,000
8,000
6,000
4,000
2,000
0
0
2
4
6
8
10
12
14
16
18
20
22
24

Calculation Example: Loan Schedule

Interest-Only Home Loan Schedule				
$1 million initial value, 3.6% pa interest rate, 30 year maturity, monthly payments				
Time months	Value $	Total payment $/month	Interest component $/month	Principal component $/month
0	1000000.00			
1	1000000.00	3000.00	3000.00	0.00
2	1000000.00	3000.00	3000.00	0.00
3	1000000.00	3000.00	3000.00	0.00
4	1000000.00	3000.00	3000.00	0.00
5	1000000.00	3000.00	3000.00	0.00
…	…	…	…	…
357	1000000.00	3000.00	3000.00	0.00
358	1000000.00	3000.00	3000.00	0.00
359	1000000.00	3000.00	3000.00	0.00
360	0.00	1003000.00	3000.00	1000000.00

$$C_{1total} = V_0 . r_{eff\ monthly,0\to1} = V_0 . \frac{r_{APR\ comp\ monthly,0\to1}}{12}$$

$$= 1{,}000{,}000 \times \frac{0.036}{12} = 3{,}000$$

$$C_{1interest} = V_0 . r_{eff\ monthly,0\to1} = V_0 . \frac{r_{APR\ comp\ monthly,0\to1}}{12}$$

$$= 1{,}000{,}000 \times \frac{0.036}{12} = 3{,}000$$

$$C_{1principal} = C_{1total} - C_{1interest}$$

$$= 3{,}000 - 3{,}000 = 0$$

Questions: Interest Only Loans

http://www.fightfinance.com/?q=29,42,57,107,160,239,298,459,

Bond Pricing exactly one period before the next coupon

The price of a bond is the present value of the coupons and the principal. The coupons are an annuity and the principal (also called the face or par value) is a single payment, therefore:

$$P_{0,bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face_T}{\left(1 + r_{eff}\right)^T}$$

Note that this equation assumes that the next coupon is paid in exactly one period. This will be the case for a bond that was just issued or a bond that just paid a coupon.

Care must be taken when choosing r_{eff} and T so that they are consistent with the time period between coupon payments (C_1).

Remember that bond yields are given as APR's compounding at the same frequency as coupons are paid, so they normally have to be converted to effective rates before using them in the above equation.

Bond Pricing Conventions

US, Japanese and Australian fixed coupon bonds often pay semi-annual coupons. Therefore the yields are quoted as APR's compounding semi-annually.

Many European bonds often pay annual coupons. Therefore the yields are quoted as APR's compounding annually, which is the same thing as an effective annual rate.

Calculation Example: Bond Pricing exactly one period before the next coupon

Question: An Australian company issues a fixed-coupon bond. The bond will mature in 3 years, has a face value of $1,000 and a coupon rate of 8% pa paid semi-annually. Yields are currently 5% pa. What is the price of the bond?

Answer: Each 6 month coupon will be:

$$C_{semi-annual} = Face\ value \times \frac{coupon\ rate}{2}$$

$$= 1{,}000 \times \frac{0.08}{2} = \$40$$

The number of time periods T must be consistent with the coupon payment frequency of 6 month periods, so

$$T = 3 \text{ years } \times 2$$

$$= 6 \text{ semi-annual periods}$$

The yield of 5% pa can be assumed to be an APR compounding every 6 months, the same frequency as the coupon payments. We need to find the effective 6 month rate to discount the 6-month coupons, so:

$$r_{eff\ 6month} = r_{APR\ comp\ semi\ annually} \div 2$$

$$= 0.05 \div 2 = 0.025$$

To find the bond price,

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

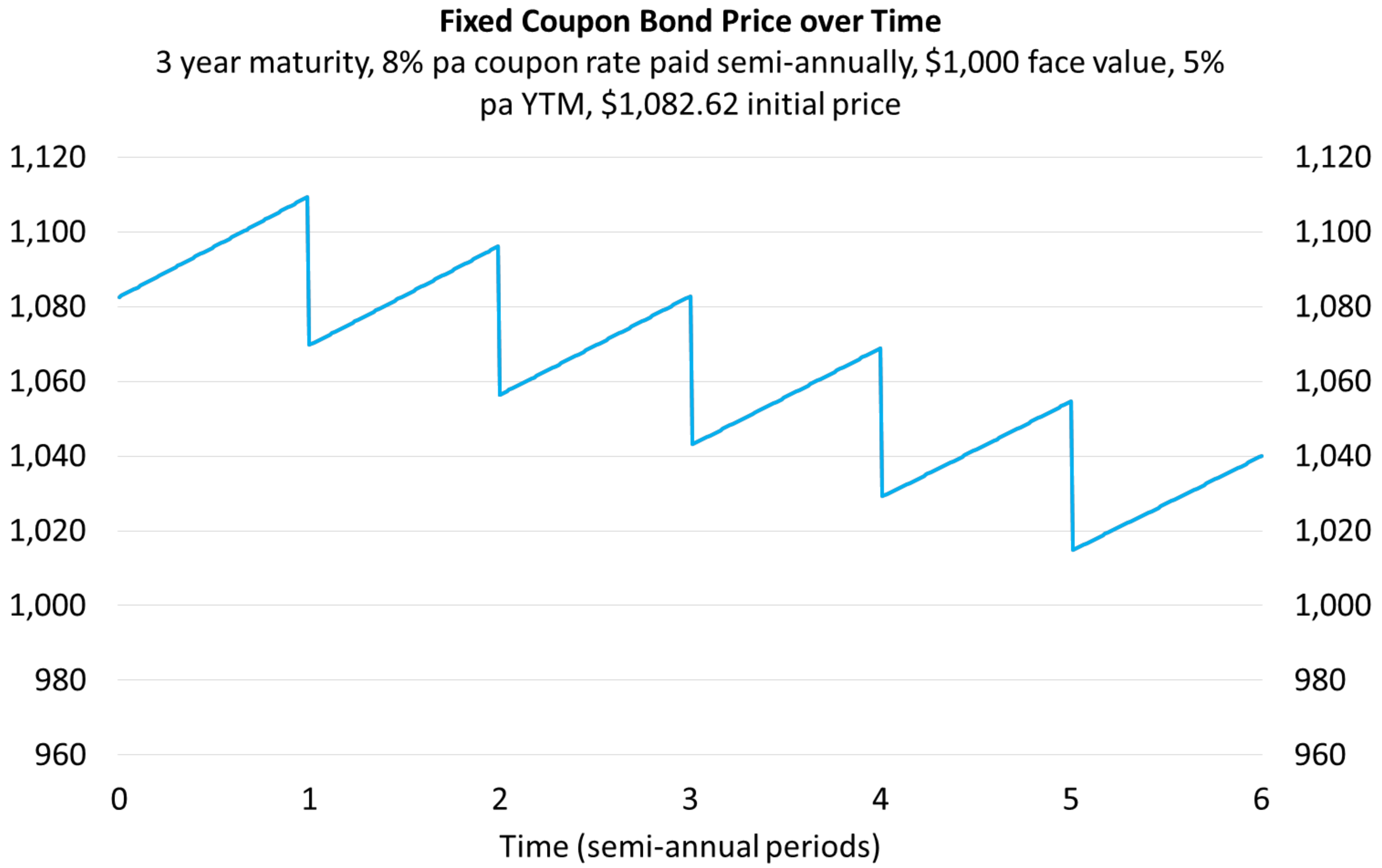
$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) \quad + \quad \frac{Face_T}{\left(1 + r_{eff}\right)^T}$$

$$= \frac{1000 \times 0.08/2}{0.05/2}\left(1 - \frac{1}{(1 + 0.05/2)^{3\times 2}}\right) + \frac{1000}{(1 + 0.05/2)^{3\times 2}}$$

$$= \frac{40}{0.025}\left(1 - \frac{1}{(1 + 0.025)^6}\right) \quad + \quad \frac{1{,}000}{(1 + 0.025)^6}$$

$$= 220.3250145 \quad + \quad 862.296866$$

$$= \$1{,}082.62$$



Fixed Coupon Bond Price over Time
3 year maturity, 8% pa coupon rate paid semi-annually, $1,000 face value, 5% pa YTM, $1,082.62 initial price
1,120
1,100
1,080
1,060
1,040
1,020
1,000
980
960
0
1
2
3
4
5
6
Time (semi-annual periods)

Bond Yields and Coupon Rates - Important

A company issues three similar bonds which differ only in their coupon rates. Coupons are paid semi-annually.

Bond	Maturity or tenor (years)	Yield to maturity (% pa)	Coupon rate (% pa)	Face or par value ($)	Price ($)	**Bond type**
A	3	5	0	100	86.23	**Discount**
B	3	5	5	100	100.00	**Par**
C	3	5	10	100	113.77	**Premium**

Discount bonds: CouponRate < YTM, Price < FaceValue

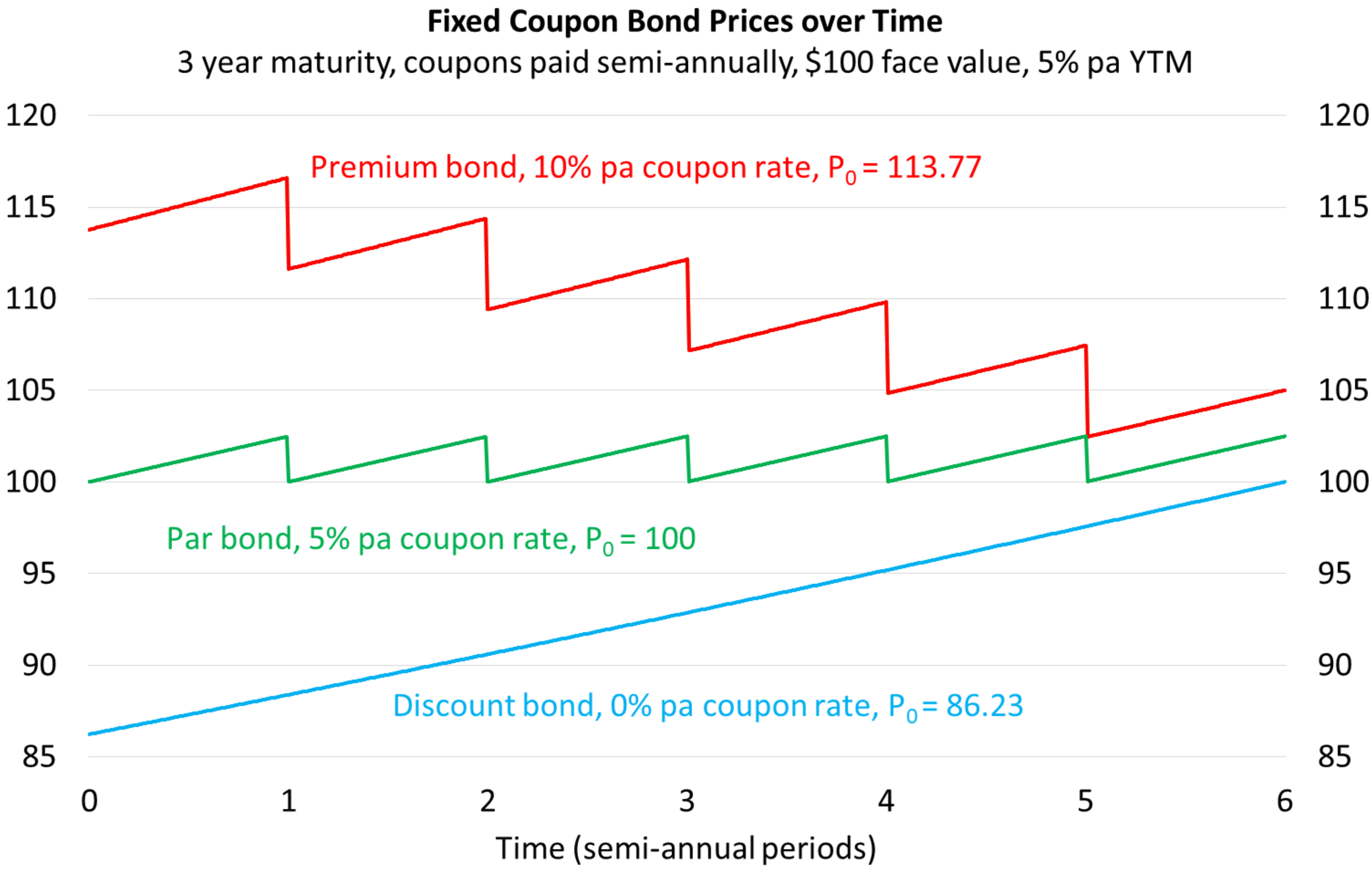
Par bonds: CouponRate = YTM, Price = FaceValue

Premium bonds: CouponRate > YTM, Price > FaceValue

P_{0A} = 100/(1+0.05/2)^(3*2) = 86.2297

P_{0B} = 100*0.05/2 * 1/(0.05/2)*(1-1/(1+0.05/2)^(2*3)) + 100/(1+0.05/2)^(3*2) = 100

P_{0C} = 100*0.1/2 * 1/(0.05/2)*(1-1/(1+0.05/2)^(2*3)) + 100/(1+0.05/2)^(3*2) = 113.7703



Fixed Coupon Bond Prices over Time
3 year maturity, coupons paid semi-annually, $100 face value, 5% pa YTM
Premium bond, 10% pa coupon rate, P0 = 113.77
Par bond, 5% pa coupon rate, P0 = 100
Discount bond, 0% pa coupon rate, P0 = 86.23
120
115
110
105
100
95
90
85
0
1
2
3
4
5
6
Time (semi-annual periods)

Calculation Example: Bonds issued at par

Question: An Australian company issues a bond at **par**. The bond will mature in 3 years, has a face value of $1,000 and a coupon rate of 8%. What is the price of the bond?

Answer: This is a trick question, no calculations are required. Since the bond was issued at par, the price must be equal to the face value. Therefore the price is $1,000. Current yields in the bond market must also be equal to 8% pa, the same as the bond's coupon rate.

Calculation Example: Zero coupon bonds

Question: An Australian company issues a zero coupon bond. The bond will mature in 3 years and has a face value of \$1,000. If the current price of the bond is \$700, what is the current yield on the bond, given as an APR compounding semi-annually?

Answer: Zero coupon bonds pay no coupons. Therefore the price of the bond is just the present value of the principal.

To find current yields we need to solve for the discount rate:

$$Price_{bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= 0 + \frac{Face}{\left(1 + r_{eff}\right)^{T}}$$

$$700 = \frac{1{,}000}{\left(1 + r_{eff\ 6mth}\right)^{6}}$$

$$700 \times \left(1 + r_{eff\ 6mth}\right)^{6} = 1{,}000$$

$$\left(1 + r_{eff\ 6mth}\right)^{6} = \frac{1{,}000}{700}$$

$$1 + r_{eff\ 6mth} = \left(\frac{1{,}000}{700}\right)^{\frac{1}{6}}$$

$$r_{eff\ 6mth} = \left(\frac{1{,}000}{700}\right)^{\frac{1}{6}} - 1$$

$$= 0.061248265$$

We need to convert this rate to an APR compounding every 6 months since that is how bond yields are quoted in Australia.

$$r_{APR\ comp\ per\ 6mths} = r_{eff\ 6mth} \times 2$$

$$= 0.061248265 \times 2$$

$$= 0.12249653 = 12.249653\%$$

For the exam, note that you will not be asked to find the yield on coupon-paying bonds since that requires trial-and-error or a computer that can do it for you (use a spreadsheet's IRR or YIELD formula).

But you may be asked to find the yield on a simple zero coupon bond like we did in this question.

Note that instead of finding the effective semi-annual rate and converting to an APR compounding semi-annually at the end, the bond pricing equation can be set up so that the APR compounding semi-annually is calculated from the start:

$$700 = \frac{1{,}000}{\left(1 + \frac{r_{APR\ comp\ 6mth}}{2}\right)^{6}}$$

$$r_{APR\ comp\ 6mth} = 2\left(\left(\frac{1{,}000}{700}\right)^{\frac{1}{6}} - 1\right)$$

$$= 2 \times 0.061248265$$

$$= 0.12249653$$

$$= 12.249653\%$$

Questions: Bond Pricing

http://www.fightfinance.com/?q=509,510,11,15,23,33,38,48,53,56,63,133,138,153,159,163,168,178,179,183,193,194,207,213,227,229,230,233,255,257,266,287,328,332,460,

Bond Pricing in between coupons

To price a bond in between coupon periods at time t, grow the bond price P_0 forward by the yield to maturity:

$P_t = P_0\left(\mathbf{1} + \boldsymbol{r_{eff}}\right)^{\boldsymbol{t}}$ where:

P_t is the bond price at the current time t and $0<t<1$;

C_1 is the next coupon payment at time one;

T is the number of coupons remaining to be paid;

r_{eff} is the yield to maturity as an effective rate per coupon period;

P_0 is the bond price one period before the next coupon C_1;

$$P_0 = \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) + \frac{F_T}{\left(1 + r_{eff}\right)^T}$$

Calculation Example: Bond pricing in between coupons

Question: A **3** year government bond paying **10**% pa semi-annual coupons with a face value of $**100** was issued **4** months ago at a yield of **5**% pa. Find the current price of the bond.

Ignore the actual number of days in each month and assume that every month is 1/12 of a year.

Answer: Let the next coupon payment in 6 months be time 1. Let's find the bond price one (semi-annual) coupon period before, which is time zero, with 6 semi-annual coupons left:

$$P_0 = \frac{0.1 \times 100/2}{0.05/2}\left(1 - \frac{1}{(1+0.05/2)^{3\times 2}}\right) + \frac{100}{(1+0.05/2)^{3\times 2}}$$

$$P_0 = \frac{0.1 \times 100/2}{0.05/2}\left(1 - \frac{1}{(1 + 0.05/2)^{3\times2}}\right) + \frac{100}{(1 + 0.05/2)^{3\times2}}$$

$$= \frac{5}{0.025}\left(1 - \frac{1}{(1 + 0.025)^{6}}\right) + \frac{100}{(1 + 0.025)^{6}}$$

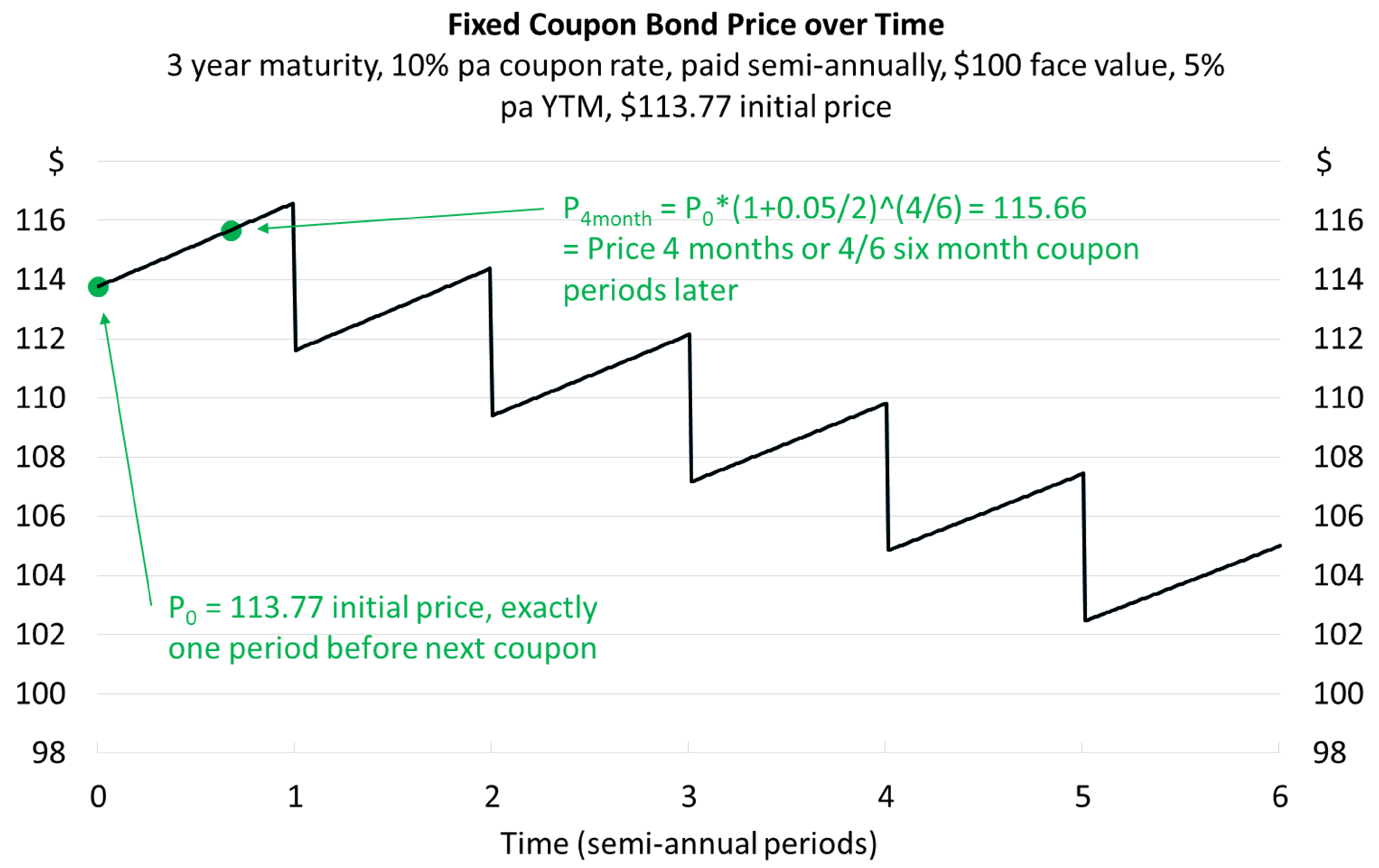
$$= 113.770313404$$

Now grow the bond price that extra 4 months forward, which is 4/6 (=0.66667) semi-annual periods, to get to the current time:

$$P_{4months} = P_0\left(1 + r_{APR\ comp\ 6\ months}/2\right)^{4/6}$$

$$= 113.770313404 \times (1 + 0.05/2)^{4/6}$$

$$= 115.6586711$$



Fixed Coupon Bond Price over Time
3 year maturity, 10% pa coupon rate, paid semi-annually, $100 face value, 5% pa YTM, $113.77 initial price
$
116
114
112
110
108
106
104
102
100
98
0
1
2
3
4
5
6
Time (semi-annual periods)
P4month = P0*(1+0.05/2)^(4/6) = 115.66
= Price 4 months or 4/6 six month coupon periods later
P0 = 113.77 initial price, exactly one period before next coupon

Calculation Example: Bond pricing in between coupons

Question: A **10** year government bond paying **3**% pa semi-annual coupons with a face value of $**100** was issued 8 months ago on 15 December 2021 at a yield of **3**% pa.

Today is 15 August 2022 and yields are now **2.8**% pa. What is the current price of the bond?

Ignore the actual number of days in each month and assume that every month is 1/12 of a year, so the bond was issued 8 months ago from today, 15 August 2022.

Answer: Let the issue date 15 December 2021 be time zero. There are only 19 semi-annual coupons left, since the first was already paid on 15 June 2022. The bond's next coupon (C_2) will be paid on 15 December 2022. The bond price one period before coupon C_2 is:

$$P_1 = \frac{C_2}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^{19}}\right) + \frac{F_{19}}{\left(1 + r_{eff}\right)^{19}} = P_{\substack{15Jun2022,\\ 6\ months\\ after\ issue}}$$

$$= \frac{0.03 \times 100/2}{0.028/2}\left(1 - \frac{1}{(1 + 0.028/2)^{19}}\right) + \frac{100}{(1 + 0.028/2)^{19}}$$

$$= 24.87274706 + 76.78543608 = 101.6581831$$

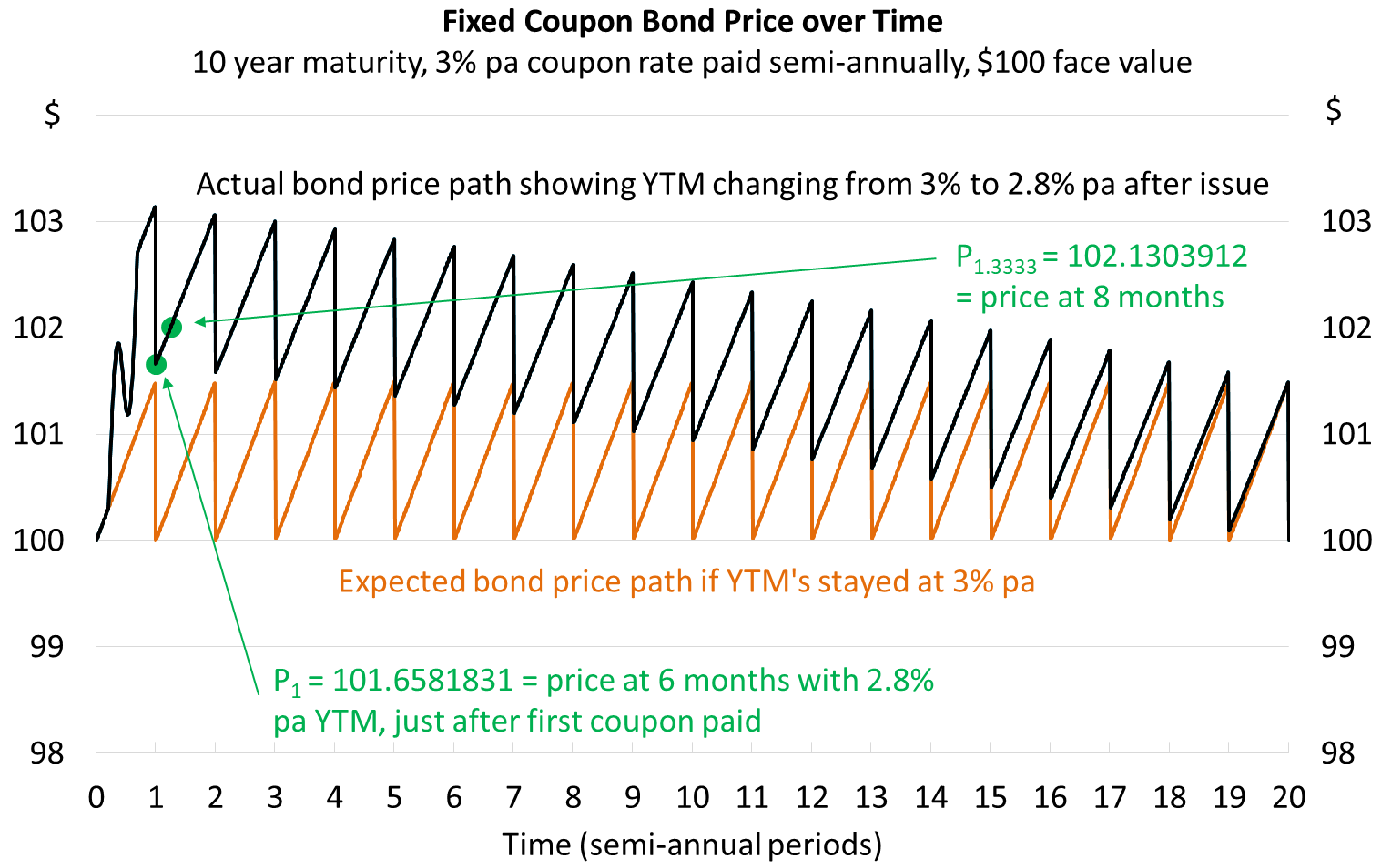
$$P_{1.3333} = P_1(1 + 0.028/2)^{0.3333} = P_{\substack{15Aug2022,\\ 8\ months\\ after\ issue}}$$

$$= 101.6581831 \times (1 + 0.028/2)^{2/6} = 102.1303912$$

The subscripts are in coupon periods so they correspond to the graph and exponents shown in the formula. So for example:

- P_0 is the initial price when the bond was issued on 15 December 2021.
- P_1 is the price 1 semi-annual period (6 months) after the bond was issued and corresponds to 15 June 2022.
- $P_{1.3333}$ is the price 1.3333 semi-annual period (8 months) after the bond was issued and corresponds to 15 August 2022. It's the current time that we're trying to price the bond.
- C_2 is the coupon 2 semi-annual periods (1 year) after the bond was issued and corresponds to 15 December 2022.

The graph helps visualize the problem.



Fixed Coupon Bond Price over Time
10 year maturity, 3% pa coupon rate paid semi-annually, $100 face value
$
$
Actual bond price path showing YTM changing from 3% to 2.8% pa after issue
$P_{1.3333}$ = 102.1303912
= price at 8 months
Expected bond price path if YTM's stayed at 3% pa
P_1 = 101.6581831 = price at 6 months with 2.8% pa YTM, just after first coupon paid
103
102
101
100
99
98
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
Time (semi-annual periods)

Debt Overview

The debt markets are far more complicated and filled with jargon than the equity markets.

It's important to be aware of all of the different ways debt can be classified. For example:

- Retail or wholesale
- Long or short term original maturity
- Floating or fixed coupons
- Secured or unsecured
- Premium, par or discount
- Call-able, put-able or vanilla
- Seniority: Senior, mezzanine, subordinated.

- **Securities** such as bonds and notes which are fungible and saleable (negotiable). **Instruments,** a broad category that encompasses securities and also loans and bills which are not fungible.
- Rated and un-rated.

Some strange things about debt markets:

- Risk is not usually quoted as a standard deviation or variance or beta, but as a 'rating'.
- Rating agencies S&P and Fitch use ratings:
    - AAA, AA+, AA, AA-, A+, A, A-, BBB+, BBB, BBB-, BB+, ...
- Moodys uses ratings:
    - Aaa, Aa1, Aa2, Aa3, A1, A2, A3, Baa1, Baa2, Baa3, Ba1, ...

- Interest rates are quoted differently depending on the market and might be given as Annualised Percentage Rates (APR's, especially in the bond market) or simple interest rates (money market).
- Debt markets trade on yields and prices based a number of days per year that could be 360 days a year, 365 days a year, or the actual days in the year, depending on the country and market.

Wholesale Debt Securities

Wholesale debt securities are traded by big financial institutions. They can be classified as being 'money market' or 'bond' securities depending on their original maturity. The Reserve Bank defines bond market debt as having an original maturity of more than one year.

Short term wholesale debt securities

Bills, commonly Bank Accepted Bills (BAB's)

Certificates of Deposit (CD's)

Promissory Notes (PN's)

Long term wholesale debt securities

Bonds, Debentures.

Short Term Wholesale Debt Securities

Short term wholesale debt securities usually have a maturity of less than 1 year when issued, such as:

- Bills, commonly Bank Accepted Bills (BAB's);
- Certificates of Deposit (CD's);
- Promissory Notes (PN's); and
- Treasury Bills (TB’s).

Yields are quoted as **simple** annual rates, which are mathematically different to compound rates such as effective rates and annualized percentage rates (APR's).

These short term debt securities do not pay coupons and are therefore 'discount securities', which means that their price is less than their face value, assuming positive yields ($r_{simple} > 0$).

$$Price_{bill} = V_0 = \frac{F_d}{\left(1 + r_{simple} \times \frac{d}{365}\right)}$$

Where F_d is the face value, r_{simple} is the simple annual interest rate and d is days until maturity.

Calculation Example: Bank Accepted Bills (BAB's)

Question: A company issues a bill which is accepted (guaranteed) by a bank. The BAB will mature in 90 days, has a face value of $1 million and an interest rate of 7% pa. What is the price of the bill?

Answer:

$$P_0 = \frac{F_d}{\left(1 + r_{simple} \times \frac{d}{365}\right)}$$

$$= \frac{1{,}000{,}000}{\left(1 + 0.07 \times \frac{90}{365}\right)} = 983{,}032.5882$$

Long-term Wholesale Debt Securities

Usually have an original maturity of more than 1 year. They're commonly known as bonds.

Yields to maturity (YTM's, often just called interest rates) are conventionally quoted as annualized percentage rates (APR's), compounding at the same frequency as the coupons are paid.

If an 8% APR compounds semi-annually, then the effective 6 month rate will be 4% per six months. You must convert APR's into effective rates before using them in the bond pricing formula:

$$P_{0,bond} = PV(annuity\ of\ coupons) + PV(principal)$$

$$= \frac{C_1}{r_{eff}}\left(1 - \frac{1}{\left(1 + r_{eff}\right)^T}\right) + \frac{Face_T}{\left(1 + r_{eff}\right)^T}$$

Most bonds pay coupons. Coupon payments are calculated as the face value multiplied by the coupon rate.

$$C_{1annual} = \mathrm{AnnualCoupon} = \mathrm{CouponRatePerAnnum} \times \mathrm{Face}_{\mathrm{T}}$$

If the coupons are paid semi-annually, then half of the total coupon is paid every 6 months.

$$C_{1\ semi\ annual} = \mathrm{SemiAnnualCoupon} = \frac{\mathrm{CouponRatePerAnnum} \times \mathrm{Face}_{\mathrm{T}}}{2}$$

Coupon rates are commonly confused with YTM's, but they are different. Coupon rates are just a convenient way to specify coupon payments.

Term Structure of Interest Rates

Long term interest rates are based on expectations of future short term interest rates.

We will discuss spot and forward rates, yield curves, and then two important theories of interest rates:

- Expectations hypothesis
- Liquidity premium theory

Another theory that we won't discuss is the 'segmented markets' theory.

Spot and Forward Interest Rates

Spot rate: An interest rate measured from now until a future time. For example, a 3-year zero-coupon bond with a yield of 8% pa has a 3-year pa spot rate of $r_{0-3,yearly} = 0.08$ pa. Note that spot rates can be from now until ***any*** future time.

Forward rate: An interest rate measured from a future time until a more distant future time. For example, if a company promised, **one year from now,** to issue a 3-year zero-coupon bond with a yield of 8% pa, then the forward rate from years 1 to 4 would be $r_{1-4,yearly} = 0.08$. Forward rates are sometimes written with an 'f' rather than 'r'.

Spot and forward rates can be quoted as APR's or effective rates.

Term Structure of Interest Rates: The Expectations Hypothesis

Expectations hypothesis is that long term spot rates (plus one) are the geometric average of the shorter term spot and forward rates (plus one) over the same time period. Mathematically:

$$1 + r_{0\to T} = \left((1 + r_{0\to 1})(1 + r_{1\to 2})(1 + r_{2\to 3})\dots\left(1 + r_{(T-1)\to T}\right)\right)^{\frac{1}{T}}$$

or

$$(1 + r_{0\to T})^{T} = (1 + r_{0\to 1})(1 + r_{1\to 2})(1 + r_{2\to 3})\dots(1 + r_{(T-1)\to T})$$

Where T is the number of periods and all rates are effective rates over each period.

Calculation Example: Term Structure of Interest Rates

Question: The following US Government Bond yields were quoted on 5/3/2012 (sourced from Bloomberg):

- 6-month zero-coupon bonds yielded 0.11%.
- 12-month zero-coupon bonds yielded 0.16%.

Find the forward rate from month 6 to 12. Quote your answer as a yield in the same form as the above yields are quoted.

Remember that US (and Australian) bonds normally pay semi-annual coupons.

Answer: Even though these are zero-coupon bonds, since they are US bonds the yield would be quoted as an APR compounding semi-annually since all coupon bonds pay semi-annual coupons. This means that our answer should be quoted in the same form, as an APR compounding semi-annually.

Therefore we have to convert the APR compounding every 6 months to an effective 6 month yield by dividing it by 2.

$$r_{0\rightarrow 0.5yr,eff\ 6mth} = \frac{r_{0\rightarrow 0.5yr,APR\ comp\ semi-annually}}{2}$$

$$= \frac{0.0011}{2} = 0.00055$$

$$r_{0\to 1yr,eff\ 6mth} = \frac{r_{0\to 1yr,APR\ comp\ semi-annually}}{2}$$

$$= \frac{0.0016}{2} = 0.0008$$

We want to find $r_{0.5yr\to 1yr,eff\ 6mth}$, which is the effective 6 month forward rate over the second 6 month period (0.5 years to 1 year).

Applying the term structure of interest rates equation:

$$(1 + r_{0\to T})^T = (1 + r_{0\to 1})(1 + r_{1\to 2})(1 + r_{2\to 3}) \ldots (1 + r_{(T-1)\to T})$$

$$\left(1 + r_{0\to 1yr,eff\ 6mth}\right)^2 = \left(1 + r_{0\to 0.5yr,eff\ 6mth}\right)\left(1 + r_{0.5\to 1yr,eff\ 6mth}\right)$$

$$(1 + 0.0008)^2 = (1 + 0.00055)\left(1 + r_{0.5\to 1yr,eff\ 6mth}\right)$$

$$r_{0.5\to1yr,eff\ 6mth} = \frac{(1+0.0008)^2}{(1+0.00055)} - 1$$

$$= 0.001050062$$

But this is an effective 6 month rate. Let's convert it to an APR compounding every 6 months.

$$r_{0.5\to1yr,APR\ comp\ 6mths} = r_{0.5\to1yr,eff\ 6mth} \times 2$$

$$= 0.001050062 \times 2$$

$$= 0.002100124 = 0.21\% pa$$

Note that this forward rate APR from 0.5 years to 1 year is bigger than both of the bond yield APR's (which are spot rates). This makes sense since we have a normal upward sloping yield curve ($r_{0\to0.5yr} < r_{0\to1yr}$) so the forward rate

$(r_{0.5 \to 1yr})$ should be greater than the spot rates $(r_{0 \to 0.5yr}\ and\ r_{0 \to 1yr})$.

Quick method: Convert rates in the term structure equation

Interest rate conversion from anualised percentage rates (APR's) to effective returns can be a headache. Most people prefer to do the APR to effective rate conversion in the expectations formula itself:

$$\left(1+r_{0\to T\ eff}\right)^{T}=\left(1+r_{0\to 1\ eff}\right)\left(1+r_{1\to 2\ eff}\right)\ldots\left(1+r_{(T-1)\to T\ eff}\right)$$

$$\left(1+r_{0\to 1yr,eff\ 6mth}\right)^{2}=\left(1+r_{0\to 0.5yr,eff\ 6mth}\right)\left(1+r_{0.5\to 1yr,eff\ 6mth}\right)$$

$$\left(1+\frac{r_{0\to 1yr,APR\ 6mths}}{2}\right)^{2}=\left(1+\frac{r_{0\to 0.5yr,APR\ 6mths}}{2}\right)\left(1+\frac{r_{0.5\to 1yr,APR\ 6mths}}{2}\right)$$

$$\left(1+\frac{0.0016}{2}\right)^{2}=\left(1+\frac{0.0011}{2}\right)\left(1+\frac{r_{0.5\to 1yr,APR\ 6mths}}{2}\right)$$

$$r_{0.5\to1yr,APR\ 6mths} = \left(\frac{\left(1+\frac{0.0016}{2}\right)^{2}}{\left(1+\frac{0.0011}{2}\right)} - 1\right) \times 2 = 0.002100124$$

This forward rate from 6 months to one year is 0.21% pa given as an APR compounding every 6 months.

Yield Curves

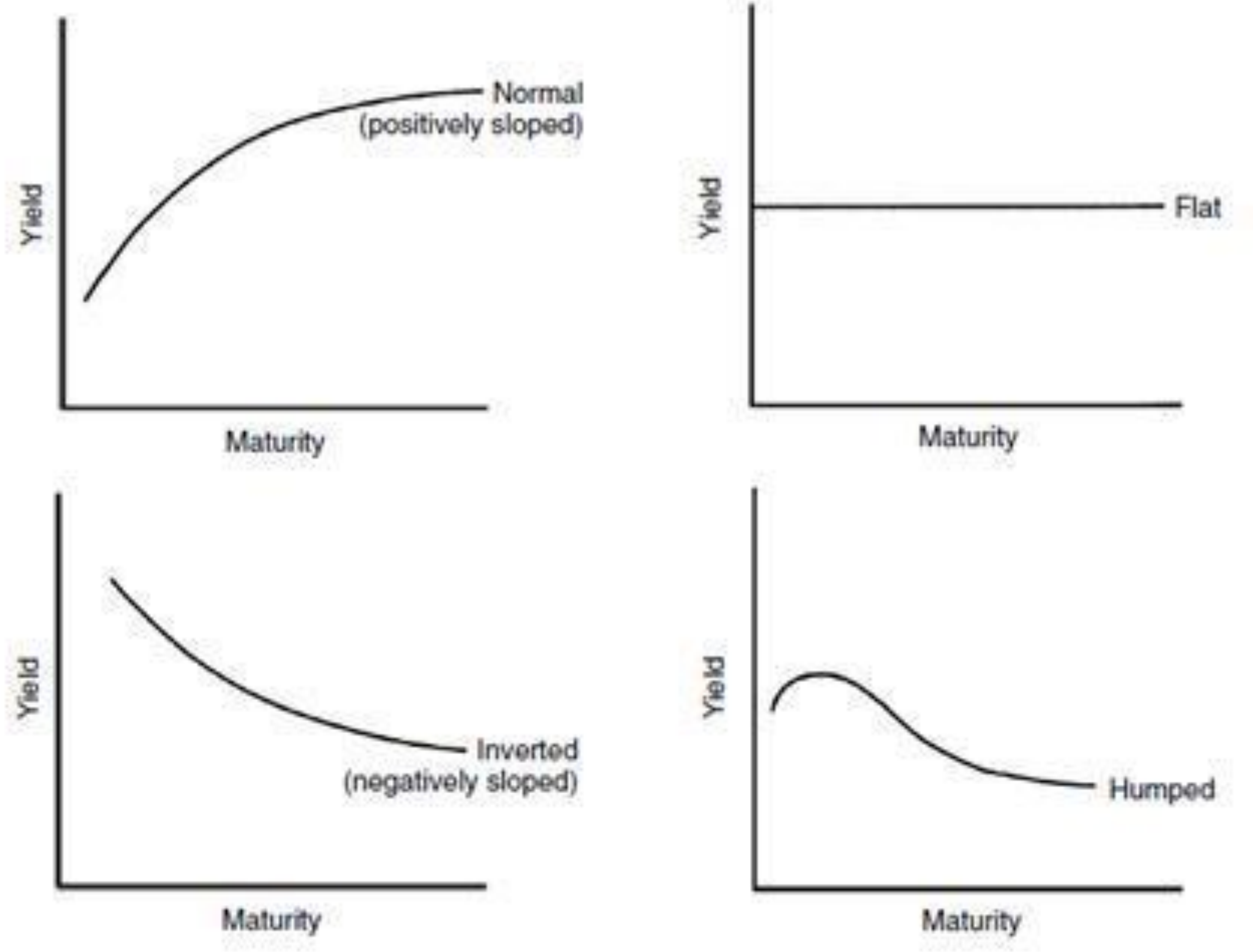
Yield curves show the behaviour of short and long-term interest rates and can give an indication of expected future interest rates.

Yield curves can be flat, normal, inverse, or some combination. The x-axis of a yield curve is the time to maturity of the bond, and the y-axis is the yield of the bond.

A **flat** yield curve is a straight horizontal line. Short and long term spot rates are equal, and yearly spot and forward rates are also equal. Other names for flat interest rates are 'constant', 'unchanging', or 'level' interest rates.

A **normal** yield curve is an upward sloping line or curve. Short term spot rates are less than long term spot rates. Yearly spot rates are less than yearly forward rates. Other names for normal yield-curves are 'upward sloping' or 'steep ' yield curves. These yield curves are 'normal' since yields usually exhibit this behaviour.

An **inverse** yield curve is a downward sloping line or curve. Short term spot rates are more than long term spot rates. Yearly spot rates are more than yearly forward rates. Other names for inverse yield-curves are 'downward sloping' or 'inverted ' yield curves.


Normal
(positively sloped)
Yield
Maturity
Flat
Yield
Maturity
Yield
Inverted
(negatively sloped)
Maturity
Yield
Humped
Maturity

An Extension: Liquidity Premium Theory

The expectations hypothesis assumes that investors are indifferent between investing in a 10 year bond, or investing in a one year bond, then investing in another 1 year bond after the first is repaid, and so on for 10 years.

Most investors would prefer to lend lots of short term bonds repeatedly rather than one big long one. The reason is that the long-term bond locks up the investor's cash and she loses the option to change her mind and do something else with the cash.

The liquidity premium theory suggests investors are only enticed to lend their cash out long-term if they are rewarded for doing so in the form of higher long-term rates compared to short term rates. This means that forward rates will be higher than the expected spot rates over the same time period.

For example, if the forward rate from years 1 to 2 is 8% now, then 1 year later the spot rate (from years 0 to 1) would tend to be less, say 7.5%.

This theory explains why the up-ward sloping yield curve is normal, since spot rates would tend to be less than forward rates.

Real World Example: Yield Curves and Term Structure of Interest Rates

See the below sources for an interesting view of yield curves and the term structure of interest rates.

Australian Federal Government (Fixed Coupon) Bond Yields:

http://www.bloomberg.com/markets/rates-bonds/government-bonds/australia

Table of yields on evening of 5/3/2012. Source: Bloomberg.

Australian Government Bonds	COUPON	MATURITY	PRICE/YIELD	PRICE/YIELD CHANGE	TIME
3-Month	0.000	06/08/2012	4.15 / 4.15	98.943 / 4.150	02/24
1-Year	6.500	05/15/2013	103.06 / 3.83	0.058 / -0.055	00:39
2-Year	6.250	06/15/2014	105.48 / 3.71	0.131 / -0.061	00:39
3-Year	6.250	04/15/2015	107.39 / 3.71	0.181 / -0.062	00:39
4-Year	4.750	06/15/2016	103.87 / 3.76	0.237 / -0.060	00:39
5-Year	6.000	02/15/2017	109.95 / 3.77	0.284 / -0.061	00:39
6-Year	5.500	01/21/2018	108.60 / 3.85	0.374 / -0.069	00:39
7-Year	5.250	03/15/2019	108.25 / 3.90	0.452 / -0.071	00:39
8-Year	4.500	04/15/2020	103.63 / 3.97	0.537 / -0.077	00:39
10-Year	5.750	05/15/2021	113.05 / 4.04	0.649 / -0.080	00:39
15-Year	4.750	04/21/2027	103.53 / 4.43	0.931 / -0.084	00:38

Orange line: current yield, Green line: previous close (yesterday's) yield. As at 5/3/2012. Note the humped curve.