

Geometric and Arithmetic Averages

$$\text{ArithmeticAverage} = x_1 \times \text{prob}_1 + x_2 \times \text{prob}_2 + \dots$$

$$\begin{aligned}\text{GeometricAverage} &= x_1^{\text{prob}_1} \times x_2^{\text{prob}_2} \times \dots \\ &= x_1^{\text{prob}_1} \times x_2^{\text{prob}_2} \times \dots\end{aligned}$$

Double-or-Nothing Coin Flip Game

Let the gambler-investor face a choice between investing completely in an investment that's a:

- Risky coin flip, paying \$2 with probability 1/2 or \$0 with probability 1/2 for each \$1 invested;
- Zero risk safe investment in cash that pays \$1 with probability 1 for each dollar invested.

	Initial wealth	Wealth if:		Arithmetic average wealth =win*probWin +loss*probLoss	Geometric average wealth =win^probWin *loss^probLoss
		Wins (50%)	Loses (50%)		
Investment:					
Risky coin flip	1	2	0	1 (=2*0.5+0*0.5)	0 (=2^0.5*0^0.5)
Safe cash	1	1	1	1 (=1*0.5+1*0.5)	1 (=1^0.5*1^0.5)

Peters' (2012) Coin Flip Game

https://www.youtube.com/watch?v=f1vXAHGlpfc&ab_channel=GreshamCollege&t=1m43s

Peter's (2012) Coin Flip Game

Let the gambler-investor face a choice between investing completely in an investment that's a:

- Risky coin flip, paying \$1.50 with probability 1/2 or \$0.60 with probability 1/2 for each \$1 invested;
- Zero risk safe investment in cash that pays \$1 with probability 1 for each dollar invested.

	Initial wealth	Wealth if:		Arithmetic average wealth =win*probWin +loss*probLoss	Geometric average wealth =win^probWin *loss^probLoss
		Wins (50%)	Loses (50%)		
Investment:					
Risky coin flip	1	1.5	0.6		
Safe cash	1	1	1		

Peters' (2012) Coin Flip Game

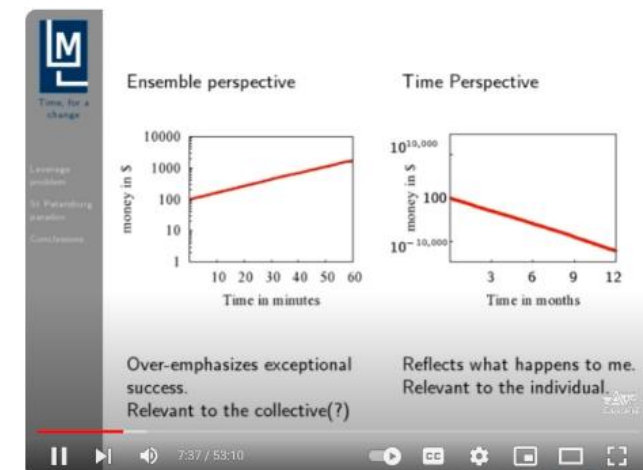
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Peter's (2012) Coin Flip Game

Let the gambler-investor face a choice between investing completely in an investment that's a:

- Risky coin flip, paying \$1.50 with probability 1/2 or \$0.60 with probability 1/2 for each \$1 invested;
- Zero risk safe investment in cash that pays \$1 with probability 1 for each dollar invested.

	Initial wealth	Wealth if:		Arithmetic average wealth	Geometric average wealth
		Wins (50%)	Loses (50%)		
Investment:					
Risky coin flip	1	1.5	0.6	1.05 $(=1.5*0.5+0.6*0.5)$	0.9487 $(=1.5^{0.5}*0.6^{0.5})$
Safe cash	1	1	1	1 $(=1*0.5+1*0.5)$	1 $(=1^{0.5}*1^{0.5})$



Arithmetic and Geometric averages: Samuelson's Gambler-Investor

Samuelson's (1971) Gambler-Investor

Let the gambler-investor face a choice between investing completely in an investment that's:

- High risk, paying \$2.70 with probability 1/2 or \$0.30 with probability 1/2 for each \$1 invested;
- Low risk, paying \$1.44 with probability 1/2 or \$0.81 with probability 1/2 for each \$1 invested; or
- Zero risk safe investment in cash that pays \$1 with probability 1 for each dollar invested.

	Initial wealth	Wealth if:		Arithmetic average wealth	Geometric average wealth
		Wins (50%)	Loses (50%)		
Investment:					
High risk investment	1	2.7	0.3		
Low risk investment	1	1.44	0.81		
Safe cash	1	1	1		

Arithmetic and Geometric averages: Samuelson's Gambler-Investor

Samuelson's (1971) Gambler-Investor

Let the gambler-investor face a choice between investing completely in an investment that's:

- High risk, paying \$2.70 with probability 1/2 or \$0.30 with probability 1/2 for each \$1 invested;
- Low risk, paying \$1.44 with probability 1/2 or \$0.81 with probability 1/2 for each \$1 invested; or
- Zero risk safe investment in cash that pays \$1 with probability 1 for each dollar invested.

	Initial wealth	Wealth if:		Arithmetic average wealth	Geometric average wealth
		Wins (50%)	Loses (50%)		
Investment:					
High risk investment	1	2.7	0.3	1.5 ($=2.7*0.5+0.3*0.5$)	0.9 ($=2.7^{0.5}*0.3^{0.5}$)
Low risk investment	1	1.44	0.81	1.125 ($=1.44*0.5+0.81*0.5$)	1.08 ($=1.44^{0.5}*0.81^{0.5}$)
Safe cash	1	1	1	1	1
