Utility Function Properties

Since rational people want **more wealth to less**, utility curves always slope upwards. After all, you can't have too much wealth. A utility curve's gradient or first derivative is always positive.

Rational people also appear to have **diminishing marginal utility from wealth**. This means that the happiness increase from receiving a fixed amount of money gets less and less as a person accumulates more wealth. So peoples' utility curves are thought to increase at a decreasing rate. Mathematically, their second derivative should always be negative. They are concave down, like a frown.

The square root function looks like this and so does the log function, so they're often used by economists to represent peoples' utility functions.

Risk Averse People

It appears that most people's increase in happiness from more and more money diminishes as they get richer.

This provides a compelling reason for risk-aversion because it means that losing $1,000 hurts more than gaining $1,000. So for example, if you had $10,000 and you gained $1,000 then that would provide less happiness than the sadness from losing $1,000 and only having $9,000 now.

People with this characteristic are called 'risk-averse'.

This is seen as normal.

It explains why people buy insurance contracts. Losing a small premium which is paid to the insurance company every month hurts less than losing your whole house in a fire and being homeless. For most people, this is true even if the present value of the insurance premiums is more than the cost of building a new house times the probability of a fire or other disaster.

Risk neutral people

People who have the same marginal happiness from every dollar they gain are risk-neutral. They will not care about risk and they're utility curve is a straight line.

Risk lovers

People who are risk-lovers will have a concave up utility curve, such as a parabola (U(W) = W^2). They like risk so much that they are willing to pay to get more. This is seen as unusual and irrational.

Calculation Example

Question: An economics teacher runs an experiment.

She approaches a poverty stricken student with zero initial wealth.

She offers him $**100** if he flips a coin and it lands on heads.

If it lands on tails he'll be paid **nothing**.

Alternatively, the poor student is offered $**30**. If he takes this certain payment, he can't take part in the single risky coin flip game.

If the student has a square root utility function, $U(W) = \sqrt{W}$, what would you expect him to do?

Flip the coin and get a risky $100 or $0, or take the certain $30?

Answer:

$$\text{ChangeInExpectedUtility} = \sum_{i=1}^{n} (Probability\ .\ UtilityChange)$$

$$= \sum_{i=1}^{2} \left(\frac{1}{2} . \left(\sqrt{\text{WealthWithCoinToss}} - \sqrt{\text{CertainWealth}} \right) \right)$$

$$= \frac{1}{2} . \left(\sqrt{100} - \sqrt{30} \right) + \frac{1}{2} . \left(\sqrt{0} - \sqrt{30} \right)$$

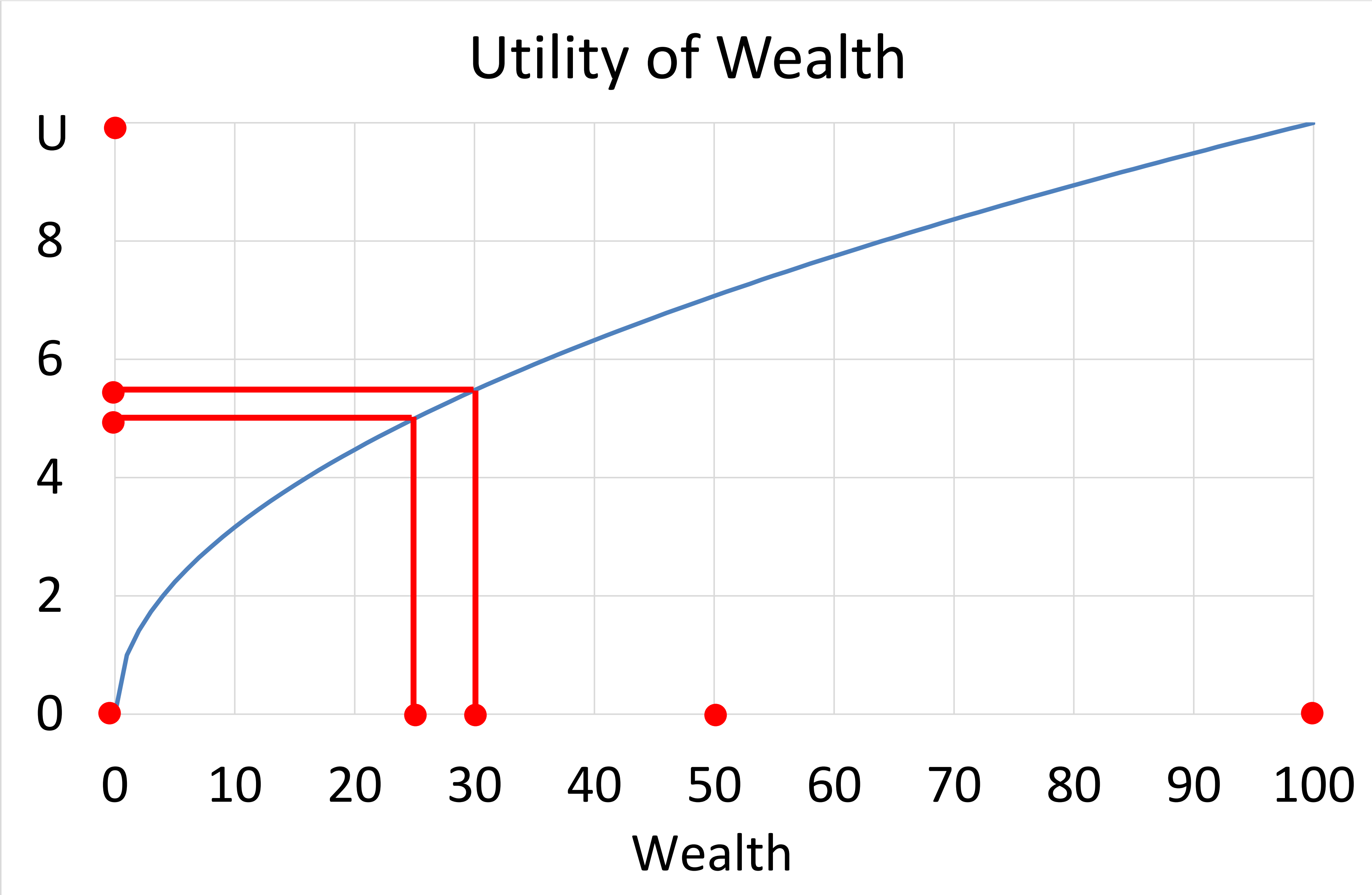
=-0.47723

Since this is a negative change in expected utility, flipping the coin is a bad idea and the student would be expected to take the certain $30 instead.

Another way of looking at it is that:

The coin flip has a utility of 5 $\left(= \frac{1}{2}.\sqrt{100} + \frac{1}{2}.\sqrt{0}\right)$; while Taking the certain $30 has a utility of 5.47723 $\left(= \frac{1}{1}.\sqrt{30}\right)$.

Therefore the certain $30 is better than the risky $100 coin flip which has a 'certainty equivalent' of $25 (=5^2).


Utility of Wealth
U
8
6
4
2
0
0
10
20
30
40
50
60
70
80
90
100
Wealth

Certainty Equivalent

The certainty equivalent of a risky gamble is the known amount of money that a person would be indifferent to having instead of taking part in the risky gamble.

For example, the certainty equivalent of the poor student in the previous question was \$25 $\left(= \left(\frac{1}{2}.\sqrt{100} + \frac{1}{2}.\sqrt{0}\right)^2\right)$ which is his utility of 5 squared since squaring utility converts it back to dollars given that the person has a square root utility function.

At a price of \$25, the student would be just as happy to flip the coin and risk getting 100 or nothing rather than taking the risk-free \$30.

Since the teacher offered $30, which is above the student's certainly equivalent, he would logically take the $30.

If the teacher offered him $20, which is below the student's certainly equivalent, he would logically take the risky coin flip.

Example: Deal or No Deal Game Show

Here is short video which helps explain the game show:

https://www.youtube.com/watch?v=hmZFHjQfx-o

A contestant who makes a surprising decision, see 3:00:

https://www.youtube.com/watch?v=H9CQscwXBt0

He can get $1 or $1m with a bank offer of around 400k and he refuses!

Question: His certainty equivalent must be higher or lower than what value?

Another surprising contestant who refuses every offer.

https://www.youtube.com/watch?v=TmWvroEQhg0

9:28

See 9:28 where $750k and $1000k are available.

See 11:07 where the bank offers $880k while the $750k and $1000k are still available.

Question: What is the expected value of the $750k and $1000k?

Question: What does the bank's offer of $880k indicate about the contestant's risk aversion, or the bank's knowledge of what's in the hidden suitcases?